DARK ENERGY, A NEW PROOF OF THE PREDICTIVE POWER OF GENERAL RELATIVITY
Stéphane Le Corre

To cite this version:
Stéphane Le Corre. DARK ENERGY, A NEW PROOF OF THE PREDICTIVE POWER OF GENERAL RELATIVITY. 2015. <ensl-01122689>

HAL Id: ensl-01122689
https://hal-ens-lyon.archives-ouvertes.fr/ensl-01122689
Submitted on 4 Mar 2015

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
DARK ENERGY, A NEW PROOF OF THE PREDICTIVE POWER OF GENERAL RELATIVITY

Stéphane Le Corre (E-mail: le.corre.stephane@hotmail.fr)
No affiliation

In a previous paper, we demonstrated that the linearized general relativity could explain dark matter (the rotation speed of galaxies, the rotation speed of dwarf satellite galaxies, the movement in a plane of dwarf satellite galaxies, the decreasing quantity of dark matter with the distance to the center of galaxies’ cluster, the expected quantity of dark matter inside galaxies and the expected experimental values of parameters \( \Omega_{dm} \) of dark matter measured in CMB). It leads, compared with Newtonian gravitation, to add a new component to gravitation without changing the gravity field (also known as gravitomagnetism). In this article we are going to see that general relativity could also explain dark energy and makes a prediction on gravitational mass of antimatter. To be consistent, this solution implies that gravitational mass of antimatter must be negative and that neutrino is not a Majorana particle. These predictions will be soon tested (AEgIS and NEMO experiments). It gives an explanation of cosmological constant and is consistent with the experimental values of parameters \( \Omega_{\Lambda} \) giving the expected order of magnitude for this cosmological constant. It predicts that dark energy (or cosmological constant) is not constant in time. Furthermore, this solution implies a cosmic inflation, leads to an explanation of the apparent disappearance of antimatter and can explain the recent acceleration of the expansion of our Universe. One also predicts several very fundamental testable results on null mass and on antimatter. The photons emitted by anti-Hydrogen should be deviated symmetrically compared to the ones emitted by Hydrogen in a gravitational field. The Lyman spectral lines of anti-Hydrogen should be shifted “symmetrically” compared to the ones of Hydrogen between two altitudes. The principle of equivalence of masses should be violated for antiprotonic helium. More prosaically, it offers an amazing image of our universe at an incredible scale.

Keywords: gravitation, gravitic field, negative mass, repulsive gravitation, cosmic inflation, dark energy, antimatter, cosmological constant, accelerating universe

1. Overview

1.1. Current solutions

Why is there a dark energy assumption? Contrary to the dark matter assumption (which is necessary from the scale of galaxies), it is at the scale of the Universe that this dark energy assumption becomes necessary. The main evidence that makes it necessary is the observation of an acceleration of the expansion of the Universe (RIESS et al., 1998; PERLMUTTER et al., 1998) in the last half of its life (SHAPIRO et al., 2005). The gravitational theories (Newtonian and general relativity) can’t explain this behavior without dark energy. From these models, a second situation reveals discrepancies between the theory and the observation. The observation of a flat Universe cannot be explained without the assumption of a dark energy (SPERGEL et al. (WMAP), 2003). Measurements on cosmic microwave background (CMB) anisotropies and baryon acoustic oscillations allow quantifying this dark energy. It represents around 70% of the energy density of the Universe (PERCIVAL et al. (WMAP), 2002). Contrary to dark matter, the dark energy exerts a negative pressure and is extremely homogeneous across the Universe.

How can we explain the origin of this dark energy? There are two kinds of theories. A first one does not modify the gravitation of general relativity. There are mainly three explanations. Models with a cosmological constant (\( \Lambda \mathrm{CDM} \) model) (GRON et al., 2007), models with new particles (quintessence, k-essence,...) (COPELAND et al., 2006) and models that consider that the observed cosmic expansion would only be a problem of “interpretation” (WILTSHIRE, 2007; ISHAK et al., 2008; TSAGAS, 2011). The second kind of theories modifies the general relativity (\( f(R) \) gravity (SOTIRIOU et al., 2010), string theory, brane cosmology (BRAX et al., 2003)....)

1.2. Solution studied in this paper

The solution presented here, does not modify general relativity. It will take into account a native term of general relativity that is, in general, neglected. But it will imply a new fundamental property of the antimatter.

This study is the logical continuation of the article on dark matter (LE CORRE, 2015). We are going to see that gravitic field (also called gravitoelectromagnetic field) appears in the Einstein equations just like the cosmological constant. And with an assumption on gravitational mass, gravitic field can then explain dark energy. We will obtain a very good order of magnitude for the cosmological constant (one of the more important result of my study). We will predict some results for current experiments (on gravitational mass for AEgIS experiment and for Majorana particle in NEMO experiment). This assumption should also lead to a cosmic inflation, an explanation of the apparent disappearance of antimatter and could explain the acceleration of the expansion of our Universe. Several predictions will be made on null mass and on antimatter that will allow testing our solution. But first, I recall the theoretical idealization used in this article. Our study will focus on the equations of linearized general relativity.
2. Gravitation in linearized general relativity

From general relativity, one deduces the linearized general relativity in the approximation of a quasi-flat Minkowski space \((g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}; |h^{\mu\nu}| \ll 1)\). With following Lorentz gauge, it gives the following field equations (HOBSON et al., 2009) (with \(\square = \frac{1}{c^2} \Box_{\mathbb{R}^3} - \Delta\)):

\[
\partial_{\mu} h^{\mu\nu} = 0; \quad \square h^{\mu\nu} = -\frac{8\pi G}{c^2} T^{\mu\nu} \quad (I)
\]

With:

\[
h^{\mu\nu} = \frac{1}{2} \eta^{\mu\nu} h; \quad h^{\mu\nu} \equiv \eta^{\mu\nu} h_{\mu\nu}; \quad \vec{h} = -h \quad (II)
\]

The general solution of these equations is:

\[
h^{\mu\nu}(ct, \vec{x}) = -\frac{4G}{c^2} \int \frac{T^{\mu\nu}(ct - \frac{1}{c} |\vec{x} - \vec{y}|, \vec{y})}{|\vec{x} - \vec{y}|} d^3\vec{y} \quad (III)
\]

In the approximation of a source with low speed, one has:

\[
T^{\mu\nu} = \rho c^2; \quad T^{0i} = c\rho u^i; \quad T^{ij} = \rho u^i u^j
\]

And for a stationary solution, one has:

\[
h^{\mu\nu}(\vec{x}) = -\frac{4G}{c^2} \int \frac{T^{\mu\nu}(\vec{y})}{|\vec{x} - \vec{y}|} d^3\vec{y}
\]

At this step, by proximity with electromagnetism, one traditionally defines a scalar potential \(\phi\) and a vector potential \(H\). There are in the literature several definitions (MASHHOON, 2008) for the vector potential \(H\). In our study, we are going to define:

\[
\begin{align*}
\phi(\vec{x}) &= -\frac{G}{c^2} \int \frac{\rho(\vec{y}) u^i(\vec{y})}{|\vec{x} - \vec{y}|} d^3\vec{y} \\
H^{i}(\vec{x}) &= \frac{G}{c^2} \int \frac{\rho(\vec{y}) u^i(\vec{y})}{|\vec{x} - \vec{y}|} d^3\vec{y}
\end{align*}
\]

With \(K\) a new constant defined by:

\[
GK = c^2
\]

This definition gives \(K^{-1} \approx 7.4 \times 10^{-28}\) very small compare to \(G\).

The field equations (I) can then be written (Poisson equations):

\[
\Delta \phi = 4\pi G \rho; \quad \Delta H^i = 4\pi G \rho u^i = 4\pi K^{-1} \rho u^i \quad (III)
\]

With the following definitions of \(\vec{g}\) (gravity field) and \(\vec{k}\) (gravitic field), those relations can be obtained from following equations:

\[
\begin{align*}
\vec{g} &= -\text{grad} \phi; \quad \vec{k} = \text{rot} \vec{H} \\
\text{rot} \vec{g} &= 0; \quad \text{div} \vec{k} = 0 \\
\text{div} \vec{g} &= -4\pi G \rho; \quad \text{rot} \vec{k} = -4\pi K^{-1} \rho
\end{align*}
\]

With relations (II), one has:

\[
h^{00} = h^{11} = h^{22} = h^{33} = \frac{2\phi}{c^2}; \quad h^{0i} = \frac{4H^i}{c}; \quad h^{ij} = 0 \quad (IV)
\]

The equations of geodesics in the linear approximation give:

\[
\frac{d^2x^i}{dt^2} = -\frac{1}{c^2} \frac{\text{grad} \phi}{c} h_{0i} + \delta^{ik}(\partial_k h_{0j} - \partial_j h_{0k}) v^j
\]

It then leads to the movement equations:

\[
\frac{d^2\vec{x}}{dt^2} = -\text{grad} \phi + 4\vec{\nabla} \text{rot} \vec{H} = \vec{g} + 4\vec{\nabla} \vec{k}
\]

From relation (IV), one deduces the metric in a quasi flat space:

\[
ds^2 = \left(1 + \frac{2\phi}{c^2}\right) c^2 dt^2 - \frac{8H_1}{c} c dt dx^i - \left(1 - \frac{2\phi}{c^2}\right) \sum (dx^i)^2
\]

In a quasi-Minkowski space, one has:

\[
H_1 dx^i = -\delta_{ij} H^j dx^i = -\vec{H} \cdot d\vec{x}
\]

We retrieve the known expression (HOBSON et al., 2009) with our definition of \(H_1\):

\[
ds^2 = \left(1 + \frac{2\phi}{c^2}\right) c^2 dt^2 - \frac{8H_1}{c} c dt dx^i - \left(1 - \frac{2\phi}{c^2}\right) \sum (dx^i)^2
\]

Remark: Of course, one retrieves all these relations starting with the parameterized post-Newtonian formalism. From (CLIFFORD M. WILL, 2014) one has:

\[
g_{0i} = -\frac{1}{2}(4\phi + 4 + 4\alpha_1) V_i; \quad V_i(\vec{x}) = \frac{G}{c^2} \int \frac{\rho(\vec{y}) u_i(\vec{y})}{|\vec{x} - \vec{y}|} d^3\vec{y}
\]

The gravitomagnetic field and its acceleration contribution are:

\[
\vec{B}_g = \vec{\nabla} \times (g_{0i} \vec{e}^i) \quad ; \quad \vec{a}_g = \vec{\nabla} \times \vec{B}_g
\]

And in the case of general relativity (that is our case):

\[
\gamma = 1; \quad \alpha_1 = 0
\]

It then gives:

\[
g_{0i} = -4V_i; \quad \vec{B}_g = \vec{\nabla} \times (-4V_i \vec{e}^i)
\]

And with our definition:

\[
H_1 = -\delta_{ij} H^j = \frac{G}{c^2} \int \frac{\rho(\vec{y}) \delta_{ij} u_i(\vec{y})}{|\vec{x} - \vec{y}|} d^3\vec{y} = V_i(\vec{x})
\]

One then has:

\[
g_{0i} = -4H_i; \quad \vec{B}_g = \vec{\nabla} \times (-4H_i \vec{e}^i) = \vec{\nabla} \times (4\delta_{ij} H^j \vec{e}^i)
\]

\[
= 4\vec{\nabla} \times \vec{H}
\]

\[
\vec{B}_g = 4\vec{\nabla} \times \vec{H}
\]

With the following definition of gravitic field:

\[
\vec{\kappa} = \frac{\vec{B}_g}{4}
\]

One then retrieves our previous relations:

\[
\vec{k} = \vec{\nabla} \times \vec{H} \quad ; \quad \vec{a}_g = \vec{\nabla} \times \vec{B}_g = 4\vec{\nabla} \times \vec{\kappa}
\]

A last remark: The interest of our notation is that the field equations are strictly equivalent to Maxwell idealization. Only the movement equations are different with the factor “4”. But of course, all the results of our study could be obtained in the traditional notation of gravitomagnetism with the relation:

\[
\vec{\kappa} = \frac{\vec{B}_g}{4}
\]

To summarize Newtonian gravitation is a traditional approximation of general relativity. But linearized general relativity shows that there is a better approximation, equivalent to Maxwell idealization in term of field equation, by adding a gravitic field very small compare to gravity field at our scale. And this approximation can also be approximated by Newtonian gravitation for many situations where gravitic field can be neglected. In other words, linearized general relativity explains how, in weak field or quasi flat space, general relativity improves Newtonian gravitation by adding a component that will become significant at the scales of galaxies.

In this approximation (linearization), the non linear terms are naturally neglected (gravitational mass is invariant and gravitation doesn’t act on itself). This approximation is valid only for low speed of source and weak field.

All these relations come from general relativity and it is in this theoretical frame that we will propose an explanation for dark energy.
3. Gravitic field: an explanation of dark energy

We are now going to see that this idealization could lead to a link with dark energy; more precisely that gravitic field can replace the cosmological constant.

3.1. Gravitic field and cosmological constant

In general relativity, to “explain” dark energy, the only way to be in agreement with experiments is to introduce with no justification a cosmological constant $\Lambda$ by using the more general Einstein’s equations (with the impulse-energy tensor $T_{kp}$ and the sign convention of (HOBSON et al., 2009):

$$G_{kp} = R_{kp} - \frac{1}{2}g_{kp}R = -\frac{8\pi G}{c^4}T_{kp} - \Lambda g_{kp}$$

Let’s write these equations in the equivalent form:

$$R_{kp} = -\frac{8\pi G}{c^4}(T_{kp} - \frac{1}{2}g_{kp}T) + \Lambda g_{kp}$$

In weak field and low speed ($T_{00} = \frac{pc^2}{2} = T$), one can write

$$-\frac{1}{2}\Delta g_{00} = -\kappa(T_{00} - \frac{1}{2}g_{00}T) + \Lambda g_{00}$$

With the traditional Newtonian approximation:

$$g_{00} = 1 + \frac{2}{c^2}\varphi$$

It gives:

$$\frac{1}{c^2}(\Delta \varphi) = \frac{8\pi G}{c^4}pc^2\left(1 - \frac{1}{2}\left(1 + \frac{2}{c^2}\varphi\right)\right) - \Lambda - \frac{2\Lambda}{c^2}\varphi$$

$$\Delta \varphi = 8\pi Gpc\left(1 - \frac{1}{c^2}\varphi\right) - c^2\Lambda - 2\Lambda \varphi$$

$$\Delta \varphi = 4\pi G\left(1 - \frac{2}{c^2}\varphi\right) - c^2\Lambda\left(1 + \frac{2}{c^2}\varphi\right)$$

In this approximation ($\frac{c^2}{\varphi} < 1$), it gives the Newtonian approximation (HOBSON et al., 2009):

$$\Delta \varphi = 4\pi Gpc - c^2\Lambda$$

(V)

What becomes this relation with our idealization? As seen in our study of the dark matter (LE CORRE, 2015), one can obtain an approximation of $g_{00}$ which contains the term $g_{0i}$:

$$g_{00} = 1 + \frac{2}{c^2}(\varphi - \theta \vec{v} \cdot \vec{H})$$

This approximation is valid in the specific configurations where the angles are around ($\vec{K} \perp \vec{F}, \vec{v} \perp \vec{K}$ and $\vec{v} \perp \vec{F}$).

So let’s use the Einstein equations without this “ad hoc” cosmological constant but with our linearized general relativity approximation:

$$\frac{1}{2}\Delta g_{00} = \kappa(T_{00} - \frac{1}{2}g_{00}T) \quad \text{and} \quad g_{00} = 1 + \frac{2}{c^2}(\varphi - \theta \vec{v} \cdot \vec{H})$$

It gives:

$$\frac{1}{c^2}(\Delta \varphi - 8\Lambda(\varphi \cdot \vec{v})) = \frac{8\pi G}{c^4}pc^2\left(1 - \frac{1}{2}\left(1 + \frac{2}{c^2}(\varphi - \theta \vec{v} \cdot \vec{H})\right)\right)$$

With the assumption of a uniform $\vec{v}$ (i.e $\partial \varphi / \partial \vec{v} \sim 0$) and with Poisson equation ($III$) ($\Delta \vec{H} = 4\pi Gpc\vec{v}$), this equation becomes:

$$\left(\Delta \varphi - 32\pi Gp\frac{\frac{1}{c^2}}{\varphi - \theta \vec{v} \cdot \vec{H}}\right) = \frac{8\pi G}{c^4}pc^2\left(1 - \frac{1}{2}\left(1 + \frac{2}{c^2}(\varphi - \theta \vec{v} \cdot \vec{H})\right)\right)$$

$$\Delta \varphi = 4\pi G\left(1 - \frac{2}{c^2}(\varphi - \theta \vec{v} \cdot \vec{H})\right) + 32\pi Gp\frac{\frac{1}{c^2}}{\varphi}$$

In our approximation ($\frac{c^2}{\varphi} < 1$), it gives the linearized general relativity approximation:

$$\Delta \varphi = 4\pi Gpc + 32\pi Gp\frac{\frac{1}{c^2}}{\varphi}$$

(VI)

By comparison with (V), the first remarkable result of this approximation is that effectively gravitic field can be equivalent to introduce a cosmological constant if $c$ is considered as constant and $p$ uniform at the scale of the Universe (traditional hypothesis). But unfortunately, the second important result is that, as is, it cannot be an explanation of dark energy. The introduction “ad hoc” of the cosmological constant is to idealize a repulsive force (a negative pressure) “$-c^2\Lambda$” to explain dark energy. Our gravitic field doesn’t have the good sign “$+32\pi Gp\frac{\frac{1}{c^2}}{\varphi}$”. Furthermore, from our previous study on dark matter (LE CORRE, 2015), this term represents the gravitic field due to ordinary matter and is in fact the term of dark matter, i.e. mainly the effects of the gravitic field of clusters of galaxies.

3.2. Gravitic field and sign of gravitational mass

We are now going to make an assumption that will allow explaining dark energy.

We just have seen that gravitic field appears in the equations just like the cosmological constant. One problem is that it hasn’t the good sign. From previous link ($32\pi Gp\frac{\frac{1}{c^2}}{\varphi}$) one can see that in our theoretical frame the only parameter that allows to obtain the good sign is the gravitational mass $\rho$.

Moreover, our approach of linearized general relativity leads to an idealization very similar to electromagnetism. But there is one fundamental difference. In electromagnetism, the charge can be negative, but gravitational mass à priori not, for experimental (and not theoretical) reasons.

These two ideas give us the temptation to see what happens if one considers a negative gravitational mass. A question is then what about inertial mass. Some studies show that if one considers a negative inertial mass, it leads to several unacceptable physical behaviors (BONDI, 1957). And certainly more important, trajectory of particles in large accelerators implies that inertial mass must be positive (we will see that negative gravitational mass should be associated to antiparticle). By consequence, we are considering in this study that the inertial mass cannot be negative. One can note that this situation is once again very similar to electromagnetism with a charge which can be negative and an inertial mass which cannot (gravitational mass can be seen as a gravitational charge).

So let’s make a fundamental assumption for our study:

Assumption (I):

- Gravitational mass ($m_g$) can be negative.
- Inertial mass ($m_i$) can only be positive.
One can note that some studies, (NI, 2003; BENOIT-LEVY et al., 2009) for examples, have ever been published with the assumption of negative mass in general relativity. But in general, in these papers, a negative inertial mass is possible to be in agreement with the principle of equivalence of masses (we will study this principle hereafter). In our study, a negative inertial mass is forbidden.

We will first focus our study on the interests of this assumption. We will see that negative gravitational mass associated with gravitic field leads to impressive results, explaining the apparent disappearance of antimatter and certainly cosmological inflation but mainly the cosmological constant $\Lambda$ and obtaining a very good order of magnitude of the quantity of dark energy. Then, we will see that this assumption implies some new fundamental consequences in the frame of general relativity. Several experiments (in particular at CERN) will soon test these consequences and therefore our solution.

### 3.3. Repulsive gravitation

A direct consequence of a negative gravitational mass and a positive inertial mass is that gravitation can be repulsive. It then leads to three consequences which could solve three expected facts that are not yet “clearly established” or explained in current theories (inflation, disappearance of antimatter and dark energy).

At this step, we need to admit that antiparticle has a negative gravitational mass (and a positive inertial mass). This fact will be demonstrated in a second part of our study (and soon tested at CERN).

#### 3.3.1. A brief cosmological story

The goal of this paragraph is only to introduce the main concepts that lead to our explanation of dark energy. More detailed studies should be done to analyze these sequences. But a roughly state of our Universe can be obtained.

We begin with the apparition of gravitation. A priori, one has three possible situations: Electromagnetism (EM) appears before gravitation (GR), EM appears at the same time that GR, EM appears after GR. Let’s make the assumption that EM appears after GR. One can note that the two others situations would certainly create particles with an electric charge and no mass. Until now, no such particle has been found, in agreement with this third situation.

Just like the idealization of the apparition of EM in current theories, we suppose that, at the “origin” of Universe, a lot of gravitational masses appear by pairs of particle and antiparticle. When GR appears (EM is not yet appeared), repulsive gravitational leads to a complex situation. Locally, positive gravitational masses (particles) will attract themselves and negative gravitational masses (antiparticles) will attract themselves generating some regions of positive gravitational masses and other regions of negative masses (segregation phase). But by the same time, these aggregations of positive gravitational masses will repulse the aggregations of negative gravitational masses and inversely (expansion/inflation phase). And these two phenomena are repeated at upper scales and everywhere in space. The evolution should lead to a complex structure, a network of positive and negative masses (a fragmented space). For example, one can imagine (like in crystallography) the following network (Fig. 1 for a 2D space, easier to represent):

![Fig. 1: Network of positive and negative gravitational masses. Each homogeneous zone (black or white disks) would lead to a universe similar to our own universe.](image)

Without simulations, it is difficult to imagine the shape of this network. But it is quite natural to imagine that the “final” structure (at the end of inflation) should be a symmetrical paving by swap between positive and negative regions (because without physical specificities due to the sign of the mass, the mathematical solution must lead to a solution that not depends on the sign of the mass). This structure should appear before EM appears which stops the expansion (EM is attractive for a pair of particle and antiparticle). At this step (end of both inflation and segregation phase), one should retrieve the “classical” history of our universe. So, à priori, these first steps can’t be seen in the cosmic microwave background (CMB). The CMB should reveal the state of our universe when this mass segregation is achieved. Finally the result of the repulsive gravitation should lead to a network which is a cluster of positive and negative regions that are separated. What we call “our Universe” is such a positive region. With this “definition” of “our Universe”, one can say that in fact these several aggregations of homogeneous masses should be the precursors of other universes. One of these “universe” particles is our Universe. So at this step we are not at the scale of our Universe but at the scale of a cluster of “universes”.

Remark on vocabulary: If we want to call this cluster “our Universe”, one then must give a new name to these regions of homogeneous masses (each ones with their own CMB and own evolution, but which are by symmetry certainly very similar). Here, I call “universe” each region of homogeneous mass (just like our current known observable Universe).

This idealization implies several consequences.

#### 3.3.2. Repulsive gravitation and dark energy

In this theoretical frame, at the scale of our Universe, one has to take into account our nearest neighbors universes which are anti-universes. Exactly like the explanation of dark matter, on the example of electromagnetism and its magnetic material, one can postulate that the network of these neighboring “universe” particles generates a non negligible external gravitic field in which the universes are embedded.

Our second assumption is then:

**Assumption (II):**
- Universe is embedded in a non negligible external gravitic field

Remark: At the scale of the cluster of universes, we have considered our Universe (and the others) as “universe” particles. It means that the gravitic field generated by the other universes on
our particle (our Universe) is represented by only one value. This approximation is compliant with the observations of a constant dark energy through our Universe (it justifies the approximation of a cosmological constant). It is notable that our theoretical solution allows explaining (or is compliant with) the constancy of dark energy. I recall that a uniform gravitic field is compliant with linearized general relativity equations (LE CORRE, 2015).

Let’s see what these nearest neighbors universes give. One can use the traditional Einstein equations without cosmological constant:

\[ G_{kp} = R_{kp} - \frac{1}{2} g_{kp} R = -\frac{8\pi G}{c^4} T_{kp} - \Lambda g_{kp} \]

We have seen previously that these equations in the linearized general relativity approximation (with \( g_{00} = 1 + \frac{z}{c^2} (\varphi - 8\ddot{v} \cdot \vec{H}) \)) give equation (VI):

\[ \Delta \varphi = 4\pi G \rho + 32\pi G \dot{\rho} \frac{v^2}{c^2} \]

Now, in our case of a universes’ cluster, our closest neighbors are anti-universes. Let’s note \( \bar{\rho} \) their mass density. Because of the symmetry of the paving by swap between positive and negative regions one can postulate that \( \bar{\rho} \sim -\rho \). Let’s note \( \sum N \) the sum on the N anti-universes around our universe and \( v_{\bar{u}} \) the speed of these universes particles. One then has \( g_{00} = 1 + \frac{z}{c^2} (\varphi - 8\ddot{v} \cdot \vec{H} - N \bar{\rho} v_{\bar{u}} \cdot \vec{H}) \). The two first terms \( (\varphi - 8\ddot{v} \cdot \vec{H}) \) concern our Universe. Let’s see the two others terms.

Just like for dark matter, the gravity fields of our nearest neighbors universe have opposite directions that should annulled the effect of gravity field \( (\sum N \varphi_{\bar{u}} = 0) \). But the gravitic fields are not opposite (and even they can be parallel). They should lead to a not null resultant gravitic field \( (\sum N \bar{H}_{\bar{u}} \neq 0) \), that is our assumption (II).

It finally gives \( g_{00} = 1 + \frac{z}{c^2} (\varphi - 8\ddot{v} \cdot \vec{H} - \sum N 8\ddot{v}_{\bar{u}} \cdot \vec{H}_{\bar{u}}) \) and the equation (VI) is then:

\[ \Delta \varphi = 4\pi G \rho + 32\pi G \dot{\rho} \frac{v^2}{c^2} + \sum N \left( 32\pi G \frac{v_{\bar{u}}^2}{c^2} \right) = 4\pi G \rho + 32\pi G \dot{\rho} \frac{v^2}{c^2} + \sum N \left( 32\pi G \frac{v_{\bar{u}}^2}{c^2} \right) \]

As seen in our study of dark matter (LE CORRE, 2015) the first term “\( 4\pi G \rho \)” represent the baryonic matter and the second “\( 32\pi G \frac{v^2}{c^2} \)” the dark matter (internal gravitic field of our Universe). We are now going to see that the third term can represent the dark energy.

**Repulsive force:**

The first point is that this term (“\( \sum N 32\pi G \frac{v_{\bar{u}}^2}{c^2} \)” has now the good sign to explain the cosmological constant. The neighborhood of anti-universes implies a repulsive force as expected by observations.

**Order of magnitude of \( \Omega_\Lambda \):**

The main consequence is that gravitic field is in same order of magnitude than expected cosmological constant, as we are going to see it now. Associated with Einstein equations with cosmological constant:

\[ G_{kp} = R_{kp} - \frac{1}{2} g_{kp} R = -\frac{8\pi G}{c^4} T_{kp} - \Lambda g_{kp} \]

One traditionally defines the two parameters:

\[ \Omega_m = \frac{8\pi G \rho}{3H^2} ; \quad \Omega_\Lambda = \frac{\Lambda}{3H^2} \]

In current theory, to be in agreement with observations, we need to make the dark matter “ad hoc” assumption that \( \rho = \rho_b + \rho_{dm} \) with \( \rho_b \) the baryonic density and \( \rho_{dm} \) the dark matter density. It then gives the three terms (\( \Omega_m = \Omega_b + \Omega_{dm} \)):

\[ \Omega_b = \frac{8\pi G \rho_b}{3H^2} ; \quad \Omega_{dm} = \frac{8\pi G \rho_{dm}}{3H^2} ; \quad \Omega_\Lambda = \frac{\Lambda}{3H^2} \]

In Newtonian approximation, they are associated with the three terms:

\[ \Delta \varphi = 4\pi G \rho - c^2 \Lambda = [4\pi G \rho_b + 4\pi G \rho_{dm}] - c^2 \Lambda \]

We want to write explicitly \( \Omega_\Lambda \) in the frame of general relativity but with our new gravitational component (i.e. without cosmological constant). For that, by comparison of the Newtonian approximation with previous cosmological parameters, one can deduce its expression. With our linearized general relativity approximation, because in our solution there is no dark matter \( (\rho_{dm} = 0) \) and then \( \rho = \rho_b \), one has (equation (VII)):  

\[ \Delta \varphi = [4\pi G \rho_b + 4\pi G \dot{\rho}_b \frac{v^2}{c^2} - \sum N (4\pi G) \rho_b \frac{v_{\bar{u}}^2}{c^2} \]

I recall that the second term is associated to dark matter \( \Omega_{dm} \) and is studied in (LE CORRE, 2015) (it leads to the relation \( \rho_{dm} \leftrightarrow \rho_b \frac{\bar{\rho} v_{\bar{u}}^2}{c^2} \)).

One then deduces that the first approximated term \( [4\pi G] \rho_b \) associated with the general relativity parameter \( \Omega_b = \frac{2\pi G \rho_b}{3H^2} \rho_b \) the second one \( [4\pi G] \rho_{dm} \) with \( \Omega_{dm} = \frac{4\pi G \rho_{dm}}{3H^2} \rho_{dm} \) and the third one \( c^2 \Lambda \) is then associated with the general relativity parameter:

\[ \Omega_\Lambda = \frac{\Lambda}{3H^2} = \sum N \left( \frac{8\pi G}{3H^2} \right) \rho_b \frac{v_{\bar{u}}^2}{c^2} \]

We can then write:

\[ \Omega_\Lambda = \Omega_b \sum N \left( \frac{8\pi G}{3H^2} \right) \rho_b \frac{v_{\bar{u}}^2}{c^2} \]

One can then obtain two important results that make our solution consistent with the observations.

First, one can deduce an order of magnitude of \( v_{\bar{u}} \), the speed of anti-universe “particles”. The parameter \( N \) represents the number of our nearest “anti-universe” particles. If we look at our previous example of “final” network in 2D one has \( N = 2^2 = 4 \). But our space is a 3D space and then with a cubic network one has \( N = 2^3 = 8 \) anti-universes around us. It gives:

\[ \Omega_\Lambda = \frac{64\pi G \rho_b \frac{v_{\bar{u}}^2}{c^2}}{3H^2} \]

The observations (PLANCK Collaboration, 2014) give \( \Omega_\Lambda \approx 0.7 \) and \( \Omega_b \approx 0.05 \). One deduces \( v_{\bar{u}} \) the speed of anti-universes:

\[ v_{\bar{u}} \approx \frac{1}{2} c \]

This result on speed of our closest universes is compliant with relativity principles and is à priori not absurd. And this value even agrees very well with what one would expect by looking at the evolution of large structures. In the following table, one has the
typical size and typical speed that characterize several large structures. At each change of scale, the radius is typically multiplied by 50 and the speed by 10:

<table>
<thead>
<tr>
<th>Galaxy</th>
<th>Cluster</th>
<th>Cluster of clusters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Typical radius (pc)</td>
<td>$5 \times 10^4$</td>
<td>$2.5 \times 10^6$</td>
</tr>
<tr>
<td>Typical speed (m/s)</td>
<td>$2 \times 10^5$</td>
<td>$2 \times 10^6$</td>
</tr>
</tbody>
</table>

Tab. 1: Typical sizes and speeds of large structures of Universe.

Remark: The value of the typical speed of cluster of clusters is in agreement with recent published results on Laniakea supercluster (BRENT TULLY et al., 2014) that give speeds until 15000 km/s.

If we continue the progression, it leads to the values:

$$r_u = 0.625 \times 10^{10} \text{pc} \sim 18 \times 10^9 \text{light-years}$$

$$v_u = 2 \times 10^9 \text{m.s}^{-1} \sim 0.6 \text{c}$$

The typical radius $r_u$ represents the size of our Universe and therefore $v_u$ would represent its typical speed. Its value is very close to the value obtained with our explanation of dark energy.

The second important result (that is similar to previous result but with another point of view) is about the order of magnitude of the cosmological constant. Our solution leads to the following explicit expression of $\Lambda$ (expression $\Lambda (VIII)$):

$$\Lambda = \frac{3H^2}{c^2} - 8\Omega_b \sum_n \frac{v_n^2}{c^2}$$

With $H = 2.3 \times 10^{-18}$ and $\Omega_b \approx 0.05$ it gives:

$$\Lambda = 0.7 \times 10^{-52} \sum_n \frac{v_n^2}{c^2}$$

With our previous $v_u \approx \frac{1}{2} \text{c}$ and $N = 8$, it gives:

$$\Lambda = 1.4 \times 10^{-52}$$

This result is in very good agreement with the expected order of magnitude of $\Lambda$. For example, the explanation of the vacuum energy density gives a cosmological constant bigger by a factor of $10^{120}$. This is the main result of our study.

4. Explanations for some facts not yet explained

We just have seen that our solution allows explaining the expected quantity of dark energy in agreement with general relativity. We are now going to see that our solution could explain the cosmic inflation, the recent acceleration of our Universe and the apparent disappearance of antimatter. It also leads to a possible evolution with time of dark energy.

4.1. Repulsive gravitation and Universe’s expansion

One of the main evidence of the existence of dark energy is the recent acceleration of our Universe expansion. Our solution can explain this fact because at each change of scale, the gravitation changes its behavior (attractive or repulsive) in function of the typical kind of mass in interaction:

- First, at the creation of pairs of mass, gravitation is repulsive between particles and anti-particles.
- Second, inside universes of homogeneous masses, gravitation is attractive.
- Third, between the « universe » particles, the gravitation is repulsive between the first close neighbors and attractive between the second close neighbors (succession of matter and antimatter).

These three steps appear successively in time because there must take more time to « build » (to make appear) a structure at an upper scale than at a lower scale (each scale “N” is built on the elements structured at the scale “N-1”). Therefore, from these three steps one can deduce three different behaviors of the expansion of the Universe with time.

4.1.1. Initial expansion (or cosmic inflation) and deceleration

Our first step leads to an initial expansion. With the momentum conservation principle, when one pair of particle and antiparticle appears, each particle of this pair escapes from the interaction of the other (EM is not yet appeared). With a lot of pairs, even if each particle has a complex trajectory, the total conservation implies that the region where all these pairs appear expands (as our previous pair of particle and antiparticle). Such an expansion could be close to the cosmic inflation postulated by some current theories. Qualitatively (and with a classical point of view), between all the possible parameters, there are two “free” initial parameters which can be adjusted to obtain the expected inflation: the distance inside a pair between the particle and the antiparticle (initial size of the pair) and the distance between the pairs (associated with density and speed of apparition of pairs). At the apparition of pairs, the effects at small scale dominate and lead to emphasize the repulsive gravitational interaction (explaining the start of the expansion). But more the distance (the dilution) increases, more the effects of the repulsive gravitational interaction decrease. At this step, large zones of homogeneous mass (without its opposite mass) appear (what I called a cluster of “universes”). And then inside these universes, the attractive gravitation becomes dominant (our second step). It should imply the stop of the inflation and a deceleration of the expansion should begin.

For a quantitative explanation, quantum gravitation is certainly necessary to simulate such a dynamical situation. But, the main result is that, in this story, an initial inflation is unavoidable because of the repulsive gravitation between particle and antiparticle.

4.1.2. Acceleration of the Universe’s expansion

The third step happens when the network of these “universe particles” begins to be well structured (cf. fig. 1). Each universe is then prisoner of a local network of opposite mass. This local network of “universe” particles (with masses of the same sign) should be in an attractive gravitational interaction and then their expansion should also decelerate. At this scale, we talk about the expansion between the universes and not inside the universes.

And if this deceleration becomes greater than the deceleration of our “inside” expansion, it means that the distance between the “universes” particles increase more slowly than the distance.
between our matters inside our Universe. Then the interaction between our “inside” matter and the “outside” antimatter increase compared to interaction between “inside” matters. And because this local network of “universes” particles and our Universe is in a repulsive gravitational interaction, it could lead to an effect of acceleration of our “inside” expansion by a compression in a perpendicular direction of our flat Universe and then an expansion along the directions of the “plane” of our Universe. Such a scenario would be in agreement with the observation of a recent acceleration of the expansion. One can note that this situation leads to see the universes particles less as spheres than flat pastilles.

4.2. Modification of the dark energy with time

Another consequence of this approach is that the gravitic field of the universes (i.e. the cosmological constant or the dark energy) should change with time. Because others universes, just like our universe, should evolve, the “final” network should evolve. It should be then in a dynamical equilibrium. So, the embedding gravitic field (our dark energy) should also evolve. At the end of the inflation, the gravitic field of the others anti-universes, felt inside our universe, should decrease with the increase of their distance to ours (due to this primordial impulsion). This is a phase of deceleration of the expansion of our Universe. But the final network should tend to stabilize this cluster of universes. One can then imagine a phenomenon of damping oscillation around a mean position of dynamical equilibrium. Concretely, our anti-universes neighbors attract themselves. But our universe stops this attraction (it is confined within the network of anti-universes). And this phenomenon is repeated for each universe particle. This oscillation of distance between universes would imply a succession of deceleration and acceleration of the expansion with certainly a damping with time.

4.3. Repulsive gravitation and disappearance of antimatter

Another consequence is that whatever the “final” structure of this network of universe particles, space should be structured with a succession of regions of positive and negative gravitational masses. And more particularly, our own Universe should have a neighborhood composed of anti-universes.

In our solution, the antiparticles have a negative gravitational mass (we will demonstrate it hereafter). The previous scenario means that each universe is composed of exclusively mainly particles or mainly antiparticles. Because this segregation phase occurs before CMB time, most antiparticles are then inaccessible from our universe. It could then explain the apparent disappearance of antimatter. One can add that our solution implies the complete symmetry between particles and antiparticles.

5. Negative gravitational mass and gravitation theories

We have seen the spectacular interests of our assumption (I) of the sign of masses by explaining (in broad terms) several unexplained experimental measures (dark energy, acceleration of our expansion, apparent disappearance of antimatter) and an expected theoretical phenomenon (cosmic inflation). We are now going to focus our study on new predictions due to this assumption (I).

First, let’s demonstrate that the current gravitational theories (Newtonian and general relativity) work well and are consistent with the assumption of the negative gravitational mass. For that, we need to write equations by distinguishing inertial and gravitational masses.

5.1. Negative gravitational mass and Newton’s laws

With $m_i$, the inertial mass (always positive), $m_g$ the gravitational mass of the test particle and $M_g$, the gravitational mass of the source, the Newtonian laws are:

$$\frac{d^2\vec{x}}{dt^2} = -G\frac{m_g M_g}{r^2}\frac{\vec{x}}{r} = -m_g\vec{\nabla}\varphi \quad (IX)$$

With $\varphi$ the gravitational potential:

$$\varphi(r) = -G\frac{M_g}{r}$$

These laws idealize the attractive force of the gravitation in our Universe $(M_g > 0$ and $m_g > 0)$.

In an anti-universe $(M_g < 0$ and $m_g < 0)$, one has:

$$\frac{d^2\vec{x}}{dt^2} = -G\frac{(-m_g)(M_g)}{r^2}\frac{\vec{x}}{r} = -G\frac{|m_g||M_g|}{r^2}\frac{\vec{x}}{r} = -|m_g|\vec{\nabla}\varphi$$

With $\varphi$ the gravitational potential:

$$\varphi(r) = -G\frac{M_g}{r}$$

The gravitational behavior in an anti-universe is then strictly equivalent to ours, attractive.

The gravitation is repulsive only for the cases $(M_g < 0$ and $m_g > 0)$ or $(M_g > 0$ and $m_g < 0)$. And this situation is consistent with Newtonian laws. There are no theoretical contradictions. The negative gravitational masses extend the Newtonian laws.

But to be completely compliant with the observations, the theory should also be consistent with the fact that, until now no repulsive gravitational interaction has been detected. Contrary to electromagnetism (which favors the mix of positive and negative charges), the repulsion between heterogeneous masses necessarily leads to a separation of the masses depending on its sign. Therefore, with our solution, the repulsive gravitational interaction allows explaining a separation of matter and antimatter and consequently the absence of the repulsive gravitation because of the apparent disappearance of the antimatter. By this way, the Newtonian laws are also consistent with the current experimental observations.

To summarize, the negative gravitational mass (with always positive inertial mass) is compliant with the Newtonian laws. It leads to two situations. In fact a third situation (on the null masses) will be studied a little further:

- Attractive gravitation: $(M_g > 0$ and $m_g > 0)$ or $(M_g < 0$ and $m_g < 0)$
- Repulsive gravitation: $(M_g > 0$ and $m_g < 0)$ or $(M_g < 0$ and $m_g > 0)$
5.2. Negative gravitational mass and general relativity

By the same way that we traditionally introduce the General relativity, for instance (HOBSON et al., 2009), we will first define an expression for the metric component $g_{00}$ obtained from geodesics’ equations and secondly we will see the consequences on the expressions of the Einstein’s equations. In a third paragraph we will see that the linearized general relativity is unchanged with the negative gravitational mass.

5.2.1. Expression of $g_{00}$

In a weak gravitational field, one has:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} ; \quad |h_{\mu\nu}| \ll 1$$

The equations of geodesics in the Newtonian approximation give:

$$\frac{d^2 \vec{x}}{dt^2} = -\frac{1}{2} c^2 h_{00} \frac{\varphi}{m_1}$$

From the equations (IX), one has:

$$\frac{d^2 \vec{x}}{dt^2} = -\frac{m_2}{m_1} \vec{\varphi}$$

One then deduces ($h_{00} = 2 \frac{m_2}{m_1} \frac{\varphi}{c^2}$):

$$g_{00} = 1 + 2 \frac{m_2}{m_1} \frac{\varphi}{c^2}$$

In an anti-universe ($M_g < 0$ and $m_g < 0$), we obtain the same expression as in our universe.

To summarize, just like for the Newtonian laws, there are the two previous situations. A third situation (on the null masses) will be studied a little further:

- ($M_g > 0$ and $m_g > 0$) or ($M_g < 0$ and $m_g < 0$):
  $$g_{00} = 1 + 2 \frac{m_g}{m_1} \frac{\varphi}{c^2}$$

- ($M_g > 0$ and $m_g < 0$) or ($M_g < 0$ and $m_g > 0$):
  $$g_{00} = 1 - 2 \frac{m_g}{m_1} \frac{\varphi}{c^2}$$

The last point is a situation that has never been observed for the same reasons (separation of the mass depending on its sign) seen previously in the case of the repulsive gravitational interaction. But it leads to several important predictions that will be soon tested. We will devote the last paragraphs to these very original consequences of our solution. But at this step, once again our assumption (I) can be seen as an extension of the current theory. One can note that our solution implies a new interpretation of general relativity because it no longer defines a single geometry of the space but at least two geometries depending on the sign of the test particle.

5.2.2. Einstein’s equations

The Einstein equations are:

$$R_{\mu\nu} = -\kappa \left( \mathcal{T}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right)$$

And the value of $\kappa$ is obtained from the Newtonian limit. One traditionally obtains $\kappa = \frac{8\pi G}{c^4}$.

What becomes $\kappa$ in our context of negative gravitational mass.

The Einstein equations can be written:

$$R_{\mu\nu} = -\kappa \left( T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right)$$

For the component $R_{00}$ and in the Newtonian approximation ($g_{00} \sim 1$ and $T_{00} = \rho_1 c^2 = T$), one obtains:

$$R_{00} = -\frac{1}{2} \kappa \rho_1 c^2$$

In this Newtonian approximation, one also has ($g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ with $|h_{\mu\nu}| \ll 1$):

$$R_{00} = -\frac{1}{2} \varphi^2 h_{00}$$

It then gives:

$$\nabla^2 h_{00} = \kappa \rho_1 c^2$$

In the Newtonian approximation, the masses can be seen as constant. So, from our previous expression:

$$g_{00} = 1 + 2 \frac{\rho_g}{\rho_1} $$

General relativity gives:

$$\nabla^2 \varphi = \frac{\kappa}{2 \rho_1} c^4$$

And from the field equations in the Newtonian gravitation:

$$\nabla^2 \varphi = 4\pi G \rho_g$$

One then deduces that:

$$k = \frac{8\pi G \rho_g}{c^4}$$

The Einstein’s equations are also consistent with the negative gravitational mass assumption and a priori they are not modified by the negative gravitational mass. This result confirms the previous conclusion that the negative gravitational mass extends the domain of validity of general relativity.

To summarize, whatever the sign of the masses $k$ is unchanged. But just like previously, a second situation (on the null masses) will be studied a little further.

5.2.3. Linearized general relativity

From previous results, one can deduce that the linearized general relativity is not modified. But let’s demonstrate it explicitly by continuing to distinguish inertial and gravitational masses.

From our value $k = \frac{8\pi G \rho_g}{c^4}$, the relations (I) become:

$$\partial_u \mathcal{H}_{\mu\nu} = 0 ; \quad \mathcal{H}_{\mu\nu} = -\frac{8\pi G \rho_g}{c^4} \frac{\mathcal{H}_{\mu\nu}}{\rho_1}$$

It gives with $\mathcal{T}_{\mu\nu} = \rho_1 u^\mu u^\nu$

$$\mathcal{H}_{\mu\nu} = -\frac{8\pi G \rho_g}{c^4} \frac{\mathcal{T}_{\mu\nu}}{\rho_1}$$

And now if we define (in agreement with $g_{00}$):

$$\mathcal{H}_0 = \frac{4\varphi}{c^2} \rho_1 ; \quad \mathcal{H}_i = \frac{4\varphi}{c^2} \frac{\mathcal{H}_i}{\rho_1} ; \quad \mathcal{H}_{ij} = 0$$

With the same definitions of $\varphi$ and $\mathcal{H}_i$ than at the beginning of our study, one obtains with ($\Delta = \frac{1}{c^2} \nabla^2 - \Delta$):

$$\varphi = 4\pi G \rho_g ; \quad \Delta \mathcal{H}_i = \frac{4\pi G}{c^4} \rho_g \mathcal{H}_i = 4\pi K^{-1} \rho_g \mathcal{H}_i$$

One effectively retrieves the equations of linearized general relativity. And as expected by our solution, it is the gravitational mass that appears in these equations allowing the negativity of these terms (our explanation of dark energy).
To summarize, with the explicitation of the Einstein’s equations in the linearized approximation, there are two situations:

- \( \rho_g > 0 \): The current known situation of the gravitation’s field.
- \( \rho_g < 0 \): Gravitation’s field with an opposite sign compared to the current known situation.

This last situation doesn’t contradict general relativity; on the contrary, it even extends the range of validity of general relativity. In fact, our explanation of the dark energy is the main prediction of general relativity in this situation. But, as said in the case of Newtonian laws, it remains to see the case of the null mass. It will be treated at the end of our study.

One can note that with our assumption (I), the linearized general relativity is completely equivalent to Maxwell idealization in term of field’s equation. Let’s use this equivalence to demonstrate that if our assumption (I) is true, then antimatter must have negative gravitational mass.

5.3. Negative gravitational mass and antmatter

As one can find in literature, (SCHIFF, 1958; CHARPIN, 1997) for examples, general relativity seems to imply that antimatter would have, if it exists, a negative mass (symmetry of the solution in the Kerr-Newman metric). By another way (because in our solution the inertial mass is always positive), we are going to demonstrate that the linearized general relativity leads also to the same conclusion. Because of its similarity with electromagnetism, we are going to use the same demonstration (ROUGE, 2005) than in electromagnetism (with traditional Klein-Gordon and Dirac equations) to show that antimatter should have negative gravitational mass:

Starting with the relativistic relationship \( E^2 = p^2c^2 + m_i^2c^4 \) (\( m_i \) the inertial mass), one obtains a quantum relation in Minkowski space from the momentum and energy operators:

\[
\hat{p} = \frac{i\hbar}{\ell} \nabla \quad \text{and} \quad \hat{E} = i\hbar \frac{\partial}{\partial t} \quad \Rightarrow \quad (\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2})\psi(t) = \frac{m_i^2c^2}{\hbar^2}\psi(t)
\]

When we want to take in account an electromagnetic field, the evolution equation can be obtained by the following substitutions:

\[
\frac{\hbar}{i} \nabla \rightarrow \frac{\hbar}{i} \nabla - q\bar{A} \quad \text{and} \quad i\hbar \frac{\partial}{\partial t} \rightarrow i\hbar \frac{\partial}{\partial t} - qV
\]

It is shown that a complex conjugation and a change of sign of the electric charge \( q \) let invariant the wave equation (solution of the evolution equation). This “conjugated” solution can be then interpreted as the idealization of the antiparticles, associated with the particle of the same inertial mass and opposite charge.

Now, let’s take into account a gravitational field. First, we show that our context of linearized general relativity is equivalent to the one of electromagnetism when the gravitational masses tend to zero.

In the approximation of linearized general relativity, we are in a quasi-flat Minkowski space, just like Klein-Gordon equations. More the gravitational masses (the field’s source and the one that undergoes it) tend to zero, more this quasi-flat Minkowski space must tend to a Minkowski space.

With our definitions, linearized general relativity is equivalent to Maxwell idealization of electromagnetism in term of field equations, as seen in the beginning of our paper, (only movement equation is different by a factor 4). Idealization of gravitation field equations, in this approximation, has then the same quadrivector \( \frac{\ell}{c^2} \bar{H} \) than electromagnetism \( \frac{\ell}{c} \bar{A} \). Once again, the masses tends to zero, more this approximation leads to this equivalence.

In the approximation of low speed, gravitational mass is invariant, just like charge in electromagnetism.

In these approximations, the only difference is in the movement equations. Gravitation "applies" a constant factor on the potential vector \( \bar{H} \) (as seen in the beginning of our study).

From all these similarities, one can then deduce that in the approximation of linearized general relativity, low speed and masses tending to zero, one has, in term of movement equations, the following correspondences \( \bar{A} \rightarrow 4\bar{H} \); \( V \rightarrow \varphi \) and \( q \rightarrow m_g \) (and same inertial mass).

Thus, in the very restricted domain of approximation of low gravitational field (quasi-flat Minkowski space) and low speed of the source, we are in a domain of validity where the idealization of gravitation is equivalent to electromagnetism. And more the masses tend to zero, better is the equivalence. So, one can apply to our linearized general relativity approximation the same previous idealization with our correspondences to extend KLEIN-GORDON equation. The evolution equation can be then obtained by the following substitutions:

\[
\frac{\hbar}{i} \nabla \rightarrow \frac{\hbar}{i} \nabla - m_g4\bar{H} \quad \text{and} \quad i\hbar \frac{\partial}{\partial t} \rightarrow i\hbar \frac{\partial}{\partial t} - m_g\varphi
\]

As previously, a complex conjugation and a change of sign of the gravitational mass \( m_g \) let invariant the equation wave. This allows showing that in the context of the existence of negative gravitational mass, antiparticles not only have opposite electric charge but also opposite gravitational mass compared to their associated particle.

We are going to see this result a little more specifically about spin \( \frac{1}{2} \) particles, from the Dirac equation. The Dirac equation is an approximation of first order of the Klein-Gordon equation

\[
i\hbar \frac{\partial}{\partial t} \psi(t) = \bar{H}_0\psi(t)
\]

With

\[
\bar{H}_0 = -i\alpha\cdot\bar{V} + m_0\beta
\]

And

\[
\alpha_1 = \begin{pmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{pmatrix} \quad \text{et} \quad \beta = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}
\]

It is of dimension 4, \( \sigma_1 \) representing Pauli matrices and \( \mathbb{I} \) the unit matrix in dimension 2.

For a charge \( q \) immersed in an electromagnetic field \( (V, \bar{A}) \), we have the Hamiltonian \( \bar{H} \):

\[
\bar{H} = \bar{a}_.(-i\bar{V} - q\bar{A}) + m_0\beta + qV
\]

Similarly, this equation can be extended to the linearized general relativity approximation (by using previous correspondences).

In our case, for a mass \( m_g \) in a gravitational field \( (\varphi, \bar{H}) \), we obtain:
\[
\hat{A} = \hat{d} \cdot (-\vec{A} - q\hat{A} - m_g\hat{A}) + m_\beta + qV + m_\phi \varphi
\]

Traditionally in the case of the only electromagnetic field, by performing the processing anti-unitary:
\[
\psi \rightarrow C\psi = U_C\psi^*
\]

With \(U_C\) a unit matrix as \(\beta U_C = -U_C\beta^*\) and \(a_1U_C = U_Ca_1^*\) (we show that we can take \(U_C = i\beta a_2\), we verify that:
\[
CH(q)C^{-1} = \hat{A}(-q)
\]
It can be applied to our new Dirac equation. Taking into account the gravitational field, we obtain:
\[
CH(m_g)C^{-1} = -\hat{A}(-m_g)
\]
And for the two fields:
\[
CH(q, m_g)C^{-1} = -\hat{A}(-q, -m_g)
\]

This result shows that an antiparticle has opposite gravitational mass and charge compared to its particle with a same inertial mass.

Remarks:
In these equations, only the gravitational mass undergoes the change of sign, in agreement with the fact that the inertial mass is always positive.

From these last equations, the result was quite remarkable that antiparticles have a negative gravitational mass. Conversely, given that so far any known object is either ordinary matter or antimatter (they are two complementary states) we can deduce that all negative gravitational mass is antimatter.

One can note that in the previous demonstration although, mathematically, one can apply the transformation \([m_g \rightarrow -m_g]\), physically this transformation makes sense only if one makes beforehand the assumption of the existence of negative gravitational masses. In other words, these equations don’t demonstrate the existence of negative gravitational masses, but if such a negative mass exists, these equations imply necessarily that it is antimatter.

**Experimental tests:**
To conclude this paragraph, our assumption (I) in the frame of general relativity implies that antiparticle must have necessarily a negative gravitational mass (and a positive inertial mass) with exactly the same opposite value than its associated particle (just like for electric charge). So antiparticle mass cannot be a slight correction of the mass of their associated particle. It leads to the following prediction:

In our solution, there is only one possibility for antimatter gravitational mass (\(\bar{m}\) means antiparticle):
\[
\bar{m}_g = -m_g
\]
All other values will mean that our solution (I) failed to explain the dark energy. And this fundamental result of our study will be soon tested.

**AEgIS and GBAR experiment at CERN:** There are some experiments at CERN that study the behavior of antimatter in a gravitational field. From our previous result, one can then deduce that, in such experiments, only one experimental result on gravitational mass is compliant with our solution \(\bar{m}_g = -m_g\)

**NEMO Experiment:** A Majorana particle is a fermion that is its own antiparticle. But with the negative gravitational mass of antiparticles, in our solution, an antiparticle with a not null gravitational mass is always different from its particle. One then cannot have a Majorana particle. It leads to another prediction:

The observation of neutrinoless double beta decay would contradict our solution. And more generally, no particle with a not null gravitational mass can be a Majorana particle (i.e. a particle that is its own antiparticle).

Remark: A recent paper (NADJ-PERGE, 2014) has published results that could be in agreement with the existence of Majorana fermions. But this experiment gives indirect evidence of a mechanism that could be explained by Majorana fermions. If this explanation is correct, then it is likely that NEMO experiment will detect Majorana particles (in contradiction with our solution). But at this stage, only a direct detection can reject our solution. It is not currently the case.

**5.4. Negative gravitational mass and principle of equivalence of masses**

The main characteristic of an interaction is that the interaction is attractive between two charges of different sign (and repulsive between two charges of same sign) or attractive between two charges of same sign (and repulsive between two charges of different sign). Therefore, this characteristic doesn’t depend on the sign of all the charges. In other words, the physical laws of interaction must be invariant whatever the arbitrary convention of the charges’ sign, just like the physical laws must be invariant whatever the referential.

In electromagnetism, the idealization of the interaction is invariant with the charge of sign of all the charges. Linearized general relativity also verifies this requirement because it is equivalent to Maxwell idealization.

Parallel to this symmetry of the theories, experiments impose, with a great accuracy, that for positive mass, one has \(m_i = m_g\).

This principle of equivalence of masses is clearly not in agreement with our assumption (I) in the case \(m_g < 0\). Our assumption (I) leads irremediably to a new principle of equivalence of masses. To continue to maintain this global symmetry (invariance with the change of sign of \(m_g\)) and to be in agreement with our assumption (I), there is only one way to extend this principle:
\[
m_i = |m_g|
\]

Furthermore, we have seen that \(\kappa = \frac{m_\phi \rho_\phi}{c^4}\) and we know that current general relativity (with \(\kappa = \frac{8\pi G}{c^4}\)) is verified with a great accuracy. It means that one must have with a great accuracy \(\frac{\rho_\phi}{\rho_i} = 1\). With our assumption (I), this relation is equivalent to have \(\rho_i = |\rho_\phi|\). By this way, for positive gravitational mass (that is the current known situation) one retrieves the same principle, meaning that the current physical results are not modified by this extension. This expression is also symmetric for the choice of the sign of gravitational mass (our invariance’s requirement for interaction). It means that this new principle is physically relevant because if one had chosen arbitrarily negative values for all our
known gravitational masses $m_g$ (choice as legitimate as the choice of positive gravitational masses) the physical theories would always be valid.

This new principle is certainly one of the most disturbing results of my study. A thing is sure, with our current knowledge, only experiments can validate or invalidate this assumption. And fortunately, as said before, we made a prediction that can soon test the validity of the assumption of a negative gravitational mass. And because of our knowledge on the trajectories of antiparticles obtained in the accelerators of particles, if negative gravitational mass is discovered, it will be impossible to maintain the same principle of equivalence (the particles’ trajectories are consistent with $\frac{m_g}{m_i}$ with only $m_i \geq 0$). So let’s see now the fundamental consequences of our new principle. It also leads to several new testable predictions.

### 5.4.1. Validity of the principle of masses’ equivalence

When we are in our positive gravitational universe, this principle is verified with a great accuracy (we will see a way to explain this fact). But with our assumption (I) this equivalence cannot be always strictly verified. We are now going to see that it should be very easy to violate this principle, leading to a new experimental test. In fact, if an object is a mixing of matter and antimatter, the equivalence of masses is always violated. This failure of the masses’ equivalence should be experimentally measurable for very simple elements that mix matter and antimatter. For example, antiprotonic helium should clearly show a great difference between inertial mass and gravitational mass.

<table>
<thead>
<tr>
<th>$m_i$</th>
<th>$m_g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3m_{ip} + 1m_{ie} + 1m_{pp} \sim 3m_{ip} + 1m_{ip} \sim 4m_{ip}$</td>
<td></td>
</tr>
<tr>
<td>$3m_{gp} + 1m_{ge} + 1m_{gp} \sim 3m_{gp} - 1m_{gp} \sim 2m_{gp}$</td>
<td></td>
</tr>
</tbody>
</table>

Due to the difficulty to maintain together matter and antimatter, one certainly has that more an object is heavy more the principle of equivalence is verified with a great accuracy (but strictly speaking, it is certainly only a very good approximation). Only “pure” objects of one kind of gravitational mass strictly verified the principle of equivalence. So, because of the repulsive interaction, it is greatly improbable that, at our scale, there are large objects with large $m_i$ and low $|m_g|$ but always objects with $m_g \sim |m_i|$ with a great accuracy. This property of gravitational interaction explains then why this principle is very well verified. But structures composed with these opposite “pure” objects are possible, just like at very large scale our cluster of universes or at very small scale the antiprotonic helium. And for these structures, the masses equivalence should be violated. A theoretical consequence is that this masses equivalence cannot be a general principle. In our solution, the masses equivalence principle would be strictly verified only in the two extremes situations:

- If one supposes that an object is composed of only positive gravitational masses, one then has $m_g = m_i$.
- If one supposes that an object is composed of only negative gravitational masses, one then has $m_g = -m_i$.

So for our previous prediction on antiprotonic helium masses, if we suppose that $m_{gp} = m_{ip}$ (i.e. that the proton and neutron are “pure” objects) one can deduce that $m_i \sim 2m_{gp}$ for antiprotonic helium.

### 5.4.2. A way explaining the masses’ equivalence principle

One can try to roughly explain the origin of this mysterious masses’ equivalence principle. Applied to electromagnetism it will also explain why the charge doesn’t follow such equivalence.

As we have already seen, we imagine the creation of masses by pairs of particle and antiparticle. For each particle, one has $m_g = cm_i$. A priori, the value of $c$ could depend on each created particle. But let us assume that this ratio $\frac{m_g}{m_i} = \alpha$ is the same for all the initial particles. With this assumption, one can finally say that we have simply transposed our principle of equivalence to the only “first” created masses. But, first we are going to see that in our solution, gravitation implies that this principle of equivalence at the “first” created particles is automatically maintain to very large scale (until our Universe’s scale). Secondly, we will justify the constancy of the initial ratio $\frac{m_g}{m_i} = \alpha$.

As we have said many times in this study, the effect of the repulsive gravitational interaction is to “purify”, to generate aggregations of homogeneous masses. This then leads naturally to maintain the ratio $\alpha$ at large scale. Indeed, if the aggregation is composed of $N$ positive gravitational masses and $M$ negative masses, with $N \gg M$ and $M \sim 0$ (due to repulsion), the gravitational mass of the object will be $M_g = Nm_g - Mm_g \sim Nm_g$. Its inertial mass will be $M_i = Nm_i + Mm_i \sim Nm_i$. One will have therefore $\frac{M_g}{M_i} = \frac{m_g}{m_i} = \alpha$. And the equality will be more accurate than the mass will be homogeneous (“pure”) whatever the mass of the object. At our scale, this result is an apparent principle of equivalence of masses (by which we choose $\alpha = 1$). The problem is now to explain how the ratio can be the same for all the created pairs. Traditionally (for electromagnetism), the creation of the pairs is idealized as a phase’s transition. We are in the same situation for gravitation. A phase’s transition is characterized by a set of well-defined values of parameters. In other words, each creation of pair is made in one specific physical context. It is quite natural to expect that at a specific physical context one has a specific physical response. In our case, it could mean that the ratio $\frac{m_g}{m_i} = \alpha$ could be relatively constant. Furthermore in our solution, an initial inflation is unavoidable. It means that at this step, the extension area, which will become our Universe, is very small making the constancy of $\alpha$ on this area more probable and it’s perhaps the main reason of this constancy.

One can apply the same procedure to define the values of electrical charges, with the difference that the ratio $\alpha = 1$ has then a physical unit. But this time, the electromagnetic interaction is attractive for opposite charges. It will tend to create, at “large” scale, neutral objects or charges either slightly positive or negative.
(slightly compare to the number of charged particles that composed the object), but unrelated to the ratio \(\alpha\). Indeed, if the object is composed of \(N\) positive electrical charges and \(M\) negative charges, with \(M \sim N\), the charge of the final object will be \(Q = (N - M)q\). Its inertial mass will be \(M_i = (N + M)m_i\).

One will have therefore \(\frac{Q}{M_i} = \frac{(N-M)}{(N+M)} \alpha \neq \alpha\). Furthermore, at the apparition of electromagnetism, the inflation decreases implying that even \(\alpha\) is less constant than for gravitation. At our scale, the result is no principle of equivalence between charges and masses.

### 5.5. Negative gravitational mass and null mass

We have seen that our assumption (I) is compliant with the current gravitation’s theories. Mainly, it extends the domain of validity of our theories in two ways. In the attractive gravitational interaction, it extends the physical behavior between negative both source and test masses. In this case, the theories are identical to our current situation of positive masses. In the repulsive gravitational interaction, it extends the physical behavior between negative and positive masses. In this case, the general relativity can explain the dark energy. But to cover all the possible cases, as announced before, a third situation must be studied, the case of the null mass. We are now going to see the consequences of this situation. It leads to an extraordinary and new fundamental consequence. But, to prepare for this consequence, let’s talk first about the problem of the existence of a null mass.

#### 5.5.1. About the null inertial mass

Physically, infinite values are not acceptable. Such values don’t imply that a theory is not mathematically consistent but can reveal the limited range of physical validity of a theory. For example, Newtonian gravitation idealization has a fundamental physical problem because its propagation speed is infinite. With linearized general relativity, one can interpret that finally this infinity appears because of an approximation which neglects the gravitic field (from its definition, if \(c \rightarrow \infty\) one has \(k \rightarrow 0\)).

In current theories of relativity, one encounters a same situation with infinite values. Basically, the limited speed appears in the change of referential with \(\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}\). And with \(v = c\), one has \(\gamma = \infty\). It means that the speed physically acceptable should be strictly inferior to \(c\). In term of contraction of lengths, a finite length should have a null length in a referential with speed \(\nu = c\), meaning that a particle with a finite length would disappear. In term of dilatation of time, it means that something would become eternal (or that time would stop). All these examples are to say that the case \(\nu = c\) is an asymptotic value that should not be physically reached.

But this limited speed is attributable to (or reachable by) only particles of null mass. Finally, the null mass is a physical value that allows reaching the unphysical speed \(c\). This situation is not satisfying, but once again, just like the infinite propagation speed of Newtonian gravitation, it does not reveal an inconsistent theory, it works very well with this physical imperfection (but it could mean that the theory is an approximation of a more accurate theory). One can also note (with our previous discussion) that the consistency of a null mass implies that such a particle has no length. Philosophically, the existence of a particle without length and without mass (even if it works mathematically) is very disturbing. And its eternity is physically unsatisfying (like any other infinity).

In quantum mechanics, a strictly null mass is also associated with a plane wave (for example a photon with a precise frequency). Such approximation implies an infinite spatial extension which is not acceptable (once again, even if it works very well in most cases).

One can also find some mathematical arguments about the domain of validity of inertial mass that corroborate the idea that a strictly null mass should only be an approximated idealization of an asymptotic situation. In an interaction, one has three possible behaviors; attraction, repulsion and neutral behavior idealized by a null value. But for an only positive characteristic (just like inertial mass) a null mass should not have a mathematical sense. One can see it just like for the algebraic structure. If one supposes that zero is a possible value, \(a + b = c\) makes always sense in a structure if there is a symmetric value (notation of group just like for an interaction). But if one supposes that value can only be positive, this equation makes always sense only in a structure that does not contain zero. Otherwise, for example, \(a + b = 0\) could be obtained (or written) but would make no sense (the structure wouldn’t be complete). Such a structure is then closer to the set of validity of the equation \(ab = c\) than \(a + b = c\) (the characteristic can be always divided but not always subtracted). Inertial mass looks like such a characteristic.

The conclusion of this discussion is that a null inertial mass, even if it works well in the current theories, is certainly more an approximated value of an infinitely small value than a strictly null value. We are going to see that our new principle with general relativity implies a specific behavior for particle of null mass that can then differentiate it from a massive particle and that can test the existence of strictly null mass.

#### 5.5.2. Negative gravitational mass and deviation of null mass

The calculation of the deviation of the particles’ trajectories in a gravitational field is based on the metric component \(g_{00}\). In our solution, one has \(g_{00} = 1 + \frac{m_y 2p}{m_i c^2}\). Let’s focus on the term \(\frac{m_y 2p}{m_i c^2}\).

We are going to look at the ratio \(\frac{m_y}{m_i}\). A priori, the value of this ratio is linked to the principle of equivalence of the masses. But to obtain our result, we do not need to use it. Inspired by the symmetry of our theories, one can say that in a constant field \(\varphi\), each time one can have theoretically a no null mass’s particle with a specific value of the ratio \(\frac{m_y}{m_i}\) one also can have theoretically an antiparticle \(\frac{m_y}{m_i}\). The term “theoretically” is only to indicate that it is a “mathematical” possibility and not an experimental requirement. Moreover with the principle of equivalence, at our scale, à priori only \(\frac{m_y}{m_i} = 1\) would be possible. From this property, one can then deduce that the term \(\frac{m_y 2p}{m_i c^2}\) is anti-symmetric with \(m_y\). What about the ratio \(\frac{m_y}{m_i}\) for the null mass? If \(m_y = 0\) one has \(\frac{m_y}{m_i} = 0\) for all values of \(m_i \neq 0\). And if \(m_i = 0\) only
\( m_g = 0 \) can have a finite limit. It only remains to find \( \lim_{m_i \to 0} \frac{m_g}{m_i} \). The symmetry of this ratio (an example is given in Fig.2) implies that the more natural extension for a null mass is \( \lim_{m_i \to 0} \frac{m_g}{m_i} = 0 \). Finally, whatever the cases, for a null mass one has the ratio \( \frac{m_g}{m_i} = 0 \).

\[
\frac{m_g}{m_i} = \frac{m_g^2 \rho}{m_i c^2} = \frac{\frac{m_g}{m_i}}{|m_g|} = 1
\]

\[
\frac{m_g}{m_i} = \frac{m_g}{m_i} = -1
\]

Fig. 2: Anti-symmetry of the ratio \( \frac{m_g}{m_i} \). To simplify the representation on this graph, we took into account the principle of equivalence but a more general representation would be a cloud of possible points distributed anti-symmetrically.

Remark: In the context of only positive masses, the more natural limit is \( \lim_{m_i \to 0} \frac{m_g}{m_i} = 1 \) in continuity with the principle of equivalence. This discrepancy will lead to a different behavior than current general relativity. While \( m_g < 0 \) extends general relativity, our solution modifies general relativity in the case \( m_g = 0 \).

One can note that for the Einstein’s equations, one obtained \( \kappa = \frac{8\pi G \rho}{c^2} \). Unlike the present case, the ratio \( \frac{m_g}{m_i} \) is natively symmetric with the change of sign of the gravitational mass. Therefore, all values \( p \) on the axis (i.e. \( \lim_{m_i \to 0} \frac{m_g}{m_i} = p \)) could be valid. In particular, our previous \( \lim_{m_i \to 0} \frac{m_g}{m_i} = 0 \) also satisfy the symmetry of \( \kappa \) in the Einstein’s equations.

From this result, one can then deduce if a particle, depending on its mass, is or not deviated in a gravitational field. Indeed, with the expression \( g_{00} = 1 + \frac{m_g}{m_i} \frac{2\rho}{c^2} \), the deviation is \( \delta c_v = \frac{m_g}{m_i} \frac{4\rho}{c^2} \). One then has:

- If \( m_g = 0 \) and \( m_i \neq 0 \) there is no deviation \( \frac{m_g}{m_i} = 0 \).
- If \( m_g = 0 \) and \( m_i = 0 \) there is no deviation (because of the anti-symmetry of the ratio \( \frac{m_g}{m_i} \)).
- If \( m_g \neq 0 \) and \( m_i \neq 0 \) there is deviation \( \frac{m_g}{m_i} \neq 0 \)

As we have just seen, if \( m_i = 0 \), the only physical value for the deviation is \( m_g = 0 \) (because otherwise \( \frac{m_g}{m_i} = \infty \)). It is then our second point. The case \( m_g \neq 0 \) and \( m_i = 0 \) is therefore impossible.

From all these cases, one can then conclude that, with our assumption (I), if \( m_g = 0 \), there is no deviation. And conversely if there is deviation, the test particle has \( m_g \neq 0 \). And even (without using the principle of equivalence) one has, if \( m_i = 0 \), there is no deviation. And conversely if there is deviation, the test particle has \( m_i \neq 0 \).

This very important result leads to several testable predictions. A first prediction is:

Until now, all particles in a gravitational field are deviated, it leads to the consequence that with our assumption (I), all known particles must have a strictly not null mass to be compliant with experimental observations. In particular, in our solution, photon must have a mass.

If we agree with the previous discussion on the irrelevance of the null inertial mass, this can be seen as an interesting enhancement of current general relativity (even if unfortunately our solution doesn’t completely prohibit this possibility). This consequence is not in contradiction with experimental test of general relativity. Indeed, the same deviation of photon can be obtained by using trajectory equations of massive particles (BOUDENOT, 1989). The situation is equivalent for the redshift.

This prediction, with our result on gravitational mass of antimatter, leads to a second one:

If the photon has a gravitational mass (even if it is infinitely small) there should have anti-photons.

One can then imagine experiments that could reveal a difference between these two frames of general relativity (with or without our assumption (I)). The deviation and spectral delay of an anti-photon should be different than for a photon. It leads to the two following possible experimental tests (we make the “relatively natural” assumption that matter generates photons and anti-matter generates anti-photons).

Deviation of photons of antimatter (anti-photons):

The traditional photon deviation expression due to gravitation \( \delta c_v = \frac{4\rho}{c^2} = \frac{4GM}{c^2v} \) is then \( \delta c_v = \frac{m_g}{m_i} \frac{4\rho}{c^2} = \frac{4GM}{c^2v} \). The deviation of an anti-photon is the same as that of a photon but with an opposite curvature. In particular, in our Universe of positive gravitational mass, photons emitted by anti-Hydrogen should be deviated symmetrically compared to the ones emitted by Hydrogen:

\[
\delta c_{\gamma} = -\delta c_v
\]

Spectral shift of photons of antimatter (anti-photons):

The traditional formula of “redshift” becomes \( 1 + z = \frac{c}{c} = \frac{\omega_{00} \omega_{00}}{\omega_{00} \omega_{00}} \). For example, in a same gravitation field of positive gravitational mass, Lyman spectral lines of anti-Hydrogen should be shifted “symmetrically” compared to the ones of Hydrogen. Instead of having \( 1 - \frac{GM}{c^2 \left( 1 - \frac{1}{v} \right) } \), one has

\[
1 + z = \frac{c}{c} = \frac{1 - \frac{GM}{c^2 \left( 1 - \frac{1}{v} \right) } \sim 1 + \frac{GM}{c^2 \left( 1 - \frac{1}{v} \right) }.
\]

Remark: If the antimatter generates anti-photons, one can wonder why one does not detect any anti-photons of the anti-universes in our Universe. In fact, the positive gravitational mass of our Universe must repel these anti-photons, just like the other objects of negative mass. Finally, these anti-photons are absent from our Universe just like the antimatter, because they are simply antimatter.
5.6. About the creation of the pairs of particle-antiparticle

For the origin of gravitational masses and electric charges, I talked about “creation” of pairs of particles. This term is certainly physically irrelevant. There are at least two reasons for this irrelevance. First, philosophically, it is always quite difficult to justify a “creation”. Strictly speaking, it means that something comes from nothing. Furthermore, mathematically, the equations never create (in the previous definition) something. As could have said Lavoisier “rien ne se perd, rien ne se crée, tout se transforme”. Secondly the creation leads to a disagreement with experiment. If in a first time, the gravitational masses are created and in a second time the charges, there should be two kinds of antimatter, one kind in our Universe and the other in the anti-universes.

![Diagram](image)

**Fig. 3:** Problem of the successive “creations” (masses then charges).

But then, in our Universe, there should be as many particles as antiparticles, in contradiction with experiment. And if we assume that the creation of masses and charges is made in the same time, the separation of the masses cannot then occur because of the attractive interaction of the electromagnetism, leading once again to have as many particles as antiparticles.

In fact, the problem can be solved by avoiding the non physical notion of “creation”. In all transitions of phase when the energy decreases, some local characteristics (that already exist but are mixed in too much energy) become effective at upper scale. In a ferromagnetic material, when the agitation decreases, the mutual influence of local spins becomes efficient and generates a transition of phase by “creating” a magnetic field at upper scale. When gas is transformed into liquid and solid, strictly speaking, nothing is created but only the local mutual links between molecules becomes more efficient than the agitation and generates a structure that becomes stable and finally rigid at upper scale. In a pictorial way, one can say that the energy of environment is like a fog that when it becomes less dense, makes appear some new unexpected things. By this way, if one assumes that, from the beginning, the particles and antiparticles exist but are in energy’s states that mask their gravitational and electromagnetic characteristics, when energy decreases, the gravitational interaction begins first to emerge. Particles and antiparticles are then separated by the repulsion gravitation (with an effect of inflation). With less energy, the electromagnetism interaction begins then to emerge. In this description, the transitions of phase are a succession of liberation of masked characteristics. By this way, our solution retrieves all its consistency.

6. Conclusion

Finally, just like we explain the dark matter of galaxies by the gravitic field of a higher structure than galaxies (cluster of galaxies), we explain the dark energy of our Universe by the gravitic field of a higher structure than our Universe (cluster of universes). But for the dark energy, a new assumption is required. We have seen that gravitic field with negative gravitational mass assumption could explain dark energy. In a funny way, with our solution, dark matter finally would reveal a new energy (the gravitic field) and dark energy a new matter (the negative gravitational mass). Of course, this solution needs to be further tested but these first results are very encouraging and right now there are some experiments that can test this theoretical frame. There are experiments (AEGIS and GBAR) at CERN that will give soon results on the gravitation of antiparticles. The solution presented here implies that antiparticle has exactly the opposite gravitational mass than its associated particle. All other behaviors won’t be in agreement with our idealization. Our solution implies too that NEMO experiment should not find evidence for the neutrinoless double beta decay. It also predicts that photons emitted by anti-Hydrogen should be deviated symmetrically compared to the ones emitted by Hydrogen in a gravitational field. By the same way, Lyman spectral lines of anti-Hydrogen should be shifted “symmetrically” compared to the ones of Hydrogen between two altitudes. And also the principle of equivalence of masses should be violated for antiprotonic helium (with $m_I \sim 2m_g$).

With this assumption on gravitational mass, gravitation leads to a more extraordinary cosmology than with only gravitic field, with a change of scale. Our own Universe could be just one little zone of positive gravitational mass lost in a set of universes and anti-universes. These anti-universes should follow the same physical law and give rise to the same symmetrical objects, anti-atoms (anti-Hydrogen...), anti-molecules (anti-water...), anti-star... and why not anti-biology. At very large scale, a universe might look like a “particle” (or more surely like a little flat pastille) with a positive or negative gravitational mass, the cluster of universe like sets of particles... The mystery of disappearance of antimatter would be also solved (it would be very far from us in anti-universes). A cosmic inflation would also be unavoidable. And, in this cluster of universes and anti-universes, interaction between these universes seems to be able to explain dark energy and the recent acceleration of the expansion of our Universe.

In current general relativity, the unicity of the geometry due the gravitation interaction allows considering that this geometry is the space geometry (and then perceived as an absolute space). Strangely, in a certain way, this absolute space plays the role of the ether (that general relativity makes it obsolete). With our solution, the new principle of equivalence of masses is not strictly a principle, but only a tendency of the repulsive gravitation to homogenize the masses. The accuracy of the principle indicates the efficiency of this homogenization. It then leads to multiple
geometries (depending on the capacity to maintain together particles and antiparticles). With our solution, general relativity doesn’t define anymore one absolute space geometry but specific geometries of the interaction entirely related to the both test particle and its environment (and not to the only environment, whatever the test particle). But because of the efficiency of the homogenization, a good approximation gives “only” three geometries and a value of $\kappa$ nearly constant with an incredible precision. Two geometries depend on the sign of the particle test and the sign of particle source ($p$) and a third geometry for null mass particle test. Then, as we have already said it, gravitational idealization is equivalent in a universe of only positive gravitational mass and in an anti-universe of only negative gravitational mass (both $p$ and $m$ change sign and then $\theta_{00}$ and $\kappa$ are unchanged). A new situation for $\theta_{00}$ (never tested until now) is when two masses of different sign are in a mutual interaction (repulsive gravitation). For this case, one can say that current general relativity is extended by our assumption (I) and dark energy is a consequence of this new theoretical situation. But, for the particle of null mass, there is a difference with current general relativity interpretation. First, the test particle follows the particular ($\kappa = 0$) Einstein’s equations $\mathcal{E}_{\mu\nu} = 0$. But furthermore, with the value of the ratio $\frac{m_\gamma}{m_\gamma} = \frac{m_\gamma}{m_\gamma} = 0$, a particle of null gravitational mass has no deviation ($\theta_{00} = 1$), implying that no strictly massless particle would be known to date.

Our approach, on the model of electromagnetism, allowed us to imagine a simplified version of a quantum mechanics of gravitation (the way to obtain the result on AEGS experiment was a first step in such a gravitational quantum mechanics). Our assumption of negative gravitational masses implies a finer adjustment of the general relativity (which becomes able to differentiate null mass). One can also wonder if the theoretical “weak point” on the null mass (in general relativity and in quantum theory) doesn’t hide a more general principle for a quantum theory of gravitation. Just like there is a physical maximal speed, there could be a physical minimal mass (such an assumption looks like a principle of quantification). In a certain way, in quantum mechanics, more a mass particle is small, more its wave appearance dominates. Strictly speaking, for a null mass it then should not have corpuscular appearance. Such a principle on a minimal mass would then explain why all objects (even the photon for example) have a wave and a corpuscular appearance.

To end, one can add that, also at very small scale, gravitation could play a stabilizing role with the negative gravitational mass. In this case, elementary particles could avoid a perpetual collapse (repulsive force). This invisible phenomenon could be a first clue to explain the strangeness of the quantum theory. Thus, gravitation could dominate the two ends of space scale and electromagnetism our intermediary scale.

References


BOUDENOT J.-C., « Electromagnétisme et gravitations relativistes », 1989 July


ISHAK et al., “Dark Energy or Apparent Acceleration Due to a Relativistic Cosmological Model More Complex than FLRW?”, arXiv:0708.2943v2, 2008 December 29

LE CORRE S., “Dark matter, a new proof of the predictive power of General Relativity”, hal-01108544v2, 2015 February


NADI-PERGE S. et al., “Observation of Majorana fermions in ferromagnetic atomic chains on a superconductor”, Science, 2014 October 2

NI G.-J., “A new insight into the negative-mass paradox of gravity and the accelerating universe”, arXiv:0308038, 2003 August 9

PERCIVAL et al., “Parameter constraints for flat cosmologies from CMB and 2dFGRS power spectra”, arXiv:0206256, 2002 August 22


SHAPIRO et al., “Do We Really Know About Cosmic Acceleration?”, arXiv:0512586, 2005 December 23

SOTIRIOU et al., “f(R) Theories Of Gravity”, arXiv:0805.1726v4, 2010 June 4

