Networks as Signals, with an Application to Bike Sharing System
Ronan Hamon, Pierre Borgnat, Patrick Flandrin, Céline Robardet

To cite this version:

HAL Id: ensl-00875187
https://hal-ens-lyon.archives-ouvertes.fr/ensl-00875187
Submitted on 21 Oct 2013

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
Networks as Signals, with an Application to a Bike Sharing System

Ronan Hamon, Pierre Borgnat, Patrick Flandrin
Physics Laboratory, CNRS UMR 5672
ENS Lyon, Université de Lyon
Lyon, France
firstname.lastname@ens-lyon.fr

Céline Robardet
LIRIS, CNRS UMR 5205
INSA Lyon, Université de Lyon
Villeurbanne, France
celine.robardet@insa-lyon.fr

Abstract—Dynamic graphs are commonly used for describing networks with a time evolution. Shimada et al. [1] recently proposed to transform these graphs into a collection of signals indexed by vertices. This approach is here further explored in a number of different directions. First, the importance of a good indexing of a graph is stressed, and a solution is proposed using a node labeling algorithm which follows the structure of the graph. Second, a spectral analysis of identified signals is performed to compute features linked to graph properties such as regularity or structure in communities. Finally, these features can be tracked over time to evidence the structure evolution of the graph. As a case study, the approach is applied to a dynamic graph based on a dataset of trips made using the bike sharing system Vélo’v in use in Lyon, France. This is shown to offer specific insights on behaviors of bike users over time in two districts of the city.

Index Terms—Dynamic networks, transportation networks, classical multidimensional scaling, cyclic bandwidth, bike sharing system

I. INTRODUCTION

Many data associated to networks, whether physical, biological or social, can be described by graphs that can become large when the data gets big. Also, the size of the data increases when the network becomes dynamic with some time evolution. These graphs are difficult to study because there exist only a few tools to describe these objects. A method to map a graph into a collection of signals has been proposed by Shimada et al. [1], based on classical multidimensional scaling (CMDS). The interest of this transformation is that it allows for a possible data reduction by restricting the expansion to a limited number of dominant contributions.

A first objective of the present work is to show how this idea can be made practical for arbitrary networks if care is taken to estimate a suitable labeling of the nodes reflecting the “bigger picture” of the network [2]. Indeed, the order in which the nodes are taken is critical to uncover information on the structure of the network using the approach described in [1]. An originality of our analysis is to adopt a signal theory approach by computing frequency analysis on signals representing the graphs. A second contribution is to show how to use the method to visualize synthetically the evolution of dynamic graphs.

Finally, an example of such a complex network evolving in time is visualized and studied thanks to the proposed approach on the basis of a dataset of trips made using the bike sharing system of Lyon, called Vélo’v.

Section II recalls the mapping from network to signals, and its shortcoming on unsorted nodes. A labeling algorithm is proposed in Section III. Then, we study the frequency analysis of the components for some examples of networks in Section IV, and show how to visualize thanks to that a simplified dynamics of a network. Section V applies the methodology to provide a simplified representation of the evolution of the Vélo’v bike sharing system. A conclusion is given in Section VI.

II. FROM NETWORKS TO SIGNALS

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ a simple undirected and unweighted graph where $\mathcal{V}$ is the set of nodes (or vertices) of size $\#\mathcal{V} = n$ and $\mathcal{E}$ is the set of links (or edges) of size $\#\mathcal{E} = m$. We note $\mathbf{A} = (a_{ij})_{i,j=1,\ldots,n}$ its adjacency matrix.

A. Method using CMDS

Shimada et al. [1] proposed a method to transform a graph into a collection of signals with $n$ points indexed by the vertices of the graph by using classical multidimensional scaling (CMDS). The interest of this transformation is based on the fact that if $n$ signals enable us to reconstruct exactly the original graph, it is also possible to reduce the representation retaining only a small number of signals, yet with a good approximation.

This transformation consists of applying CMDS on a matrix distance between vertices of a graph, noted $\Delta = (\delta)_{i,j=1,\ldots,n}$ and defined for $i, j \in \mathcal{V}$ by

$$\delta_{ij} = \begin{cases} 0 & \text{if } i = j \\ 1 & \text{if } a_{ij} = 1 \text{ and } i \neq j \\ w > 1 & \text{if } a_{ij} = 0 \text{ and } i \neq j \end{cases}$$

Following [1], we choose $w = 1.1$.

Multidimensional scaling (MDS) [3] is a set of mathematical techniques used to represent measurements of similarity (or dissimilarity) among pairs of objects as distances between points in a multidimensional space whose dimension is low. Classical MDS is a particular case of metric MDS where the dissimilarities are assumed to be Euclidean distances.
The matrix $X$ of coordinates in the low-dimensional, transformed space can be computed analytically. Starting with the distance matrix $\Delta$, we first compute a double centering of the matrix whose terms are squared : $B = -\frac{1}{2}J\Delta^2J$ with $J = I_n - \frac{1}{n}1_n1_n^T$ where $I_n$ is the identity matrix and $1_n1_n^T$ an $n \times n$ matrix of ones.

The CMDS solution is given by $X = Q+\Lambda^\frac{1}{2}$ with $\Lambda$ a diagonal matrix whose terms are the eigenvalues of the matrix $B$ sorted in an increasing order and $Q_+$ is the matrix of the corresponding eigenvectors. The obtained signals are the components (or columns) of the matrix $X$ and the $j$-th signal is noted $X^{(j)}$. These signals are indexed by the vertices of the graph.

B. The necessity of ordering labels of nodes

Shimada et al. show that ring lattices are transformed to periodic signals : each component is a cosine whose frequency depends on the number of the components. More generally, it can be relevant to describe signals using spectral analysis and hence link frequency features with graph properties. Nevertheless, the spectral analysis is closely related to the indexation of signals and so to the numbering of the graph. It becomes necessary to find a good numbering of nodes in order to have therefore good spectral properties. We propose a method based on the fact that we want to avoid brutal variations of signals. That means that it is necessary to have close indices between neighbor vertices, which are defined closer in the distance matrix than unlinked vertices. We propose in the next section an algorithm to relabel nodes such that the indices between neighbor nodes are close. In particular, abrupt variations of signals are to be avoided. Figure 1 shows examples of transformation for two graphs with 100 nodes. The first one is a Watts-Strogatz graph with degree $k = 4$ and no probability of relinkage ($p = 0$) [4] (Figure 1a) which is transformed into cosines if the label of nodes follows the cycle. A random labeling gives signals with high variability. The second example consists of the transformation of a graph with 3 communities (Figure b1). The first two components highlight the community structure of the graph when the labeling browses nodes by community. In both graphs, when the labeling is random, the signals have a high variability.

III. ALGORITHM OF NODE LABELING

The labeling of a graph with $n$ nodes assigns to each node an integer between 0 and $n - 1$. One possible labeling that will have the property advocated above comes from the cyclic bandwidth sum problem [5] that can be formalized as follows. Let $d$ be a distance between two labels $i$ and $j$ defined by $d(i, j) = \min(|i - j|, n - |i - j|)$. Solving the cyclic bandwidth problem consists of finding a one-to-one and onto mapping $\lambda$ of $\mathcal{V}$ to $\{0, \ldots, n - 1\}$ defined by

$$\hat{\lambda} = \min_\lambda f(\lambda) \quad \text{with} \quad f(\lambda) = \sum_{\{u,v\} \in \mathcal{E}} d(\lambda(u), \lambda(v))$$

We propose to solve this problem thanks to a two-step algorithm. The first step is based on the depth-first search algorithm and enables us to obtain a collection of independent paths. An approximation of the longest path starting from node $u$ is computed using the following procedure, where $u$ is the node with highest eccentricity [4] :

Ensure: $P$ a path
1: NextNode $\leftarrow u$
2: while NextNode $\neq \emptyset$ do
3: \hspace{1em} NextNode $\leftarrow \emptyset$
4: \hspace{1em} Push $u$ in $P$
5: \hspace{1em} for all $v$ such that $(u, v) \in \mathcal{E}$, $v \notin P$ do
6: \hspace{2em} if degree of $v = 1$ then
7: \hspace{3em} \hspace{1em} Push $v$ in $P$
8: \hspace{2em} else
9: \hspace{3em} \hspace{1em} if NextNode $= \emptyset$ or Topological_Similarity(u,v) > Topological_Similarity(u, NextNode) then
10: \hspace{4em} \hspace{1em} NextNode $\leftarrow v$
11: end if
12: end if
13: end for
14: end while

The algorithm goes on until all the paths have been inserted.\(^1\)

\(^1\)The full description of this algorithm can be found at http://perso.ens-lyon.fr/ronan.hamon/files/relabeling.pdf.
Figure 2 shows the labeling of a 2d grid graph before and after nodes relabeling : the labeling follows the structure of the graph.

IV. FREQUENCY ANALYSIS OF NETWORK’S SIGNALS

Let us consider a collection of $K$ signals indexed by $n$ vertices. Several features can be computed to describe the components. We note $S(k, f) = |\mathcal{F}X(k)(f)|^2$ which is estimated, for positive frequencies, on $\frac{n}{2} + 1$ bins, $\forall k \in \{1, \cdots, K\}$, $\mathcal{F}$ being the Fourier transform. We then compute the following features :

1) Energy of components:
\[
\forall k \in \{1, \cdots, K\}, E_k = \|X(k)\|^2
\]

2) Energy of frequencies:
\[
\forall f \in \{1, \cdots, \frac{n}{2} + 1\}, \tilde{S}(f) = \sum_{k=1}^{K} S(k, f)
\]

Figure 3 shows the frequency analysis corresponding to graphs introduced in section II, evidencing that specific frequencies patterns can be linked to graph properties such as regularity or presence of communities.

Extension to dynamic graphs

Spectral analysis of signals derived from a transformation of a graph enables us to highlight relations between feature patterns and graph properties. This approach can be extended to dynamic graphs : considering that at each time step a graph snapshot can be generated where the vertices are retained nodes corresponding to the stations belonging either to the 7th district of Lyon or to the city of Villeurbanne. The study is hence focused on the relation between these geographical areas which are not contiguous but close enough to enable short distance trips by bike. To determine if there are areas which can be considered as two communities, i.e. in this context an area where users who rent a bike tend to stay, a normalized averaged distance in labeling between the stations is computed. This criterion is equal to zero if there is zero distance, nonzero if there are some areas which are well separated. The analysis of the energy of frequencies in Figure 4c over all components is less obvious. It shows nonetheless the presence of high energy of low frequencies in the first few components which indicates the presence of hidden structures inside the graph as suggested in Figure 3b. In order to highlight these structures, a subgraph of the Vélo’v graph based on geographical consideration is generated and analyzed in the next section.

B. Study of a subgraph

A subgraph is generated from the Vélo’v graph where the retained nodes correspond to the stations belonging either to the 7th district of Lyon or to the city of Villeurbanne. The study is hence focused on the relation between these two geographical areas which are not contiguous but close enough to enable short distance trips by bike. To determine if there are areas which can be considered as two communities, i.e. in this context an area where users who rent a bike tend to stay, a normalized averaged distance in labeling between the two areas is computed. This criterion is equal to zero if all edges between communities are separated. The opposite occurs at 9pm where the value is maximal, meaning that the areas are not separated by labeling. The study is focused on the relation between these two areas and is maximal, meaning that the two areas are well separated. The study is focused on the relation between these two areas and is maximal, meaning that the two areas are well separated. Figures 5a and 5b illustrate this results by displaying colors of nodes using geographical positions, highlighting the fact that geographical areas can be seen as communities at 9pm. Finally the spectra of graphs enable us to find a structure in communities as described in section IV at time 9pm while the much less concentrated energy at time 10am suggests no such organization.
VI. CONCLUSION

A signal theory approach has been proposed for studying dynamic graphs. It has been shown to offer new ways of highlighting key features in time evolving networks, and in particular in social networks such as bike sharing systems. Paving the way for further extensions and investigations, the present study is expected to contribute to a better understanding of urban rhythms and, more generally, of the structure evolution over time of complex networks in other domains.

ACKNOWLEDGEMENT

This work is supported by the programs ARC 5 and ARC 6 of the region Rhône-Alpes.

REFERENCES