

Functionalized AFM probes for force spectroscopy: eigenmodes shape and stiffness calibration through thermal noise measurements

Justine Laurent, Audrey Steinberger, Ludovic Bellon

► **To cite this version:**

Justine Laurent, Audrey Steinberger, Ludovic Bellon. Functionalized AFM probes for force spectroscopy: eigenmodes shape and stiffness calibration through thermal noise measurements. Nanotechnology, Institute of Physics, 2013, 24, pp.225504. 10.1088/0957-4484/24/22/225504. ensl-00787242v2

HAL Id: ensl-00787242

<https://hal-ens-lyon.archives-ouvertes.fr/ensl-00787242v2>

Submitted on 6 May 2013

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Functionalized AFM probes for force spectroscopy: eigenmodes shape and stiffness calibration through thermal noise measurements

Justine Laurent, Audrey Steinberger and Ludovic Bellon*

Université de Lyon, Laboratoire de Physique

École Normale Supérieure de Lyon, CNRS

46 allée d'Italie, FR 69007, Lyon, France

(Dated: May 6, 2013)

Supplementary data

Tables I to V report numerical values of $\alpha_n(\tilde{m}, \tilde{r})$, for the first 5 modes, $0 \leq \tilde{m} \leq 2$ and $0 \leq \tilde{r} \leq 0.1$.

| | | Mode 1: $\alpha_1(\tilde{m}, \tilde{r})$ | | | | | | | | | | |
|-------------|--------|--|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| | | \tilde{r} | | | | | | | | | | |
| | | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | 0.10 |
| \tilde{m} | 0.00 | 1.8751 | 1.8751 | 1.8751 | 1.8751 | 1.8751 | 1.8751 | 1.8751 | 1.8751 | 1.8751 | 1.8751 | 1.8751 |
| | 0.10 | 1.7227 | 1.7227 | 1.7226 | 1.7225 | 1.7223 | 1.7221 | 1.7218 | 1.7215 | 1.7211 | 1.7207 | 1.7203 |
| | 0.20 | 1.6164 | 1.6164 | 1.6162 | 1.6161 | 1.6158 | 1.6155 | 1.6151 | 1.6146 | 1.6140 | 1.6134 | 1.6127 |
| | 0.30 | 1.5361 | 1.5361 | 1.5360 | 1.5357 | 1.5354 | 1.5350 | 1.5345 | 1.5340 | 1.5333 | 1.5325 | 1.5317 |
| | 0.40 | 1.4724 | 1.4724 | 1.4722 | 1.4720 | 1.4716 | 1.4712 | 1.4707 | 1.4700 | 1.4693 | 1.4685 | 1.4676 |
| | 0.50 | 1.4200 | 1.4199 | 1.4198 | 1.4195 | 1.4191 | 1.4187 | 1.4181 | 1.4175 | 1.4167 | 1.4158 | 1.4149 |
| | 0.60 | 1.3757 | 1.3756 | 1.3755 | 1.3752 | 1.3748 | 1.3744 | 1.3738 | 1.3731 | 1.3723 | 1.3714 | 1.3704 |
| | 0.70 | 1.3375 | 1.3374 | 1.3373 | 1.3370 | 1.3366 | 1.3362 | 1.3356 | 1.3349 | 1.3341 | 1.3332 | 1.3321 |
| | 0.80 | 1.3041 | 1.3040 | 1.3039 | 1.3036 | 1.3032 | 1.3027 | 1.3021 | 1.3014 | 1.3006 | 1.2997 | 1.2987 |
| | 0.90 | 1.2745 | 1.2744 | 1.2742 | 1.2740 | 1.2736 | 1.2731 | 1.2725 | 1.2718 | 1.2710 | 1.2700 | 1.2690 |
| | 1.00 | 1.2479 | 1.2479 | 1.2477 | 1.2474 | 1.2470 | 1.2465 | 1.2459 | 1.2452 | 1.2444 | 1.2435 | 1.2424 |
| | 1.10 | 1.2239 | 1.2239 | 1.2237 | 1.2234 | 1.2230 | 1.2225 | 1.2219 | 1.2212 | 1.2204 | 1.2195 | 1.2184 |
| | 1.20 | 1.2021 | 1.2020 | 1.2018 | 1.2016 | 1.2012 | 1.2007 | 1.2001 | 1.1994 | 1.1986 | 1.1976 | 1.1966 |
| | 1.30 | 1.1820 | 1.1820 | 1.1818 | 1.1815 | 1.1812 | 1.1807 | 1.1801 | 1.1794 | 1.1785 | 1.1776 | 1.1766 |
| | 1.40 | 1.1636 | 1.1635 | 1.1633 | 1.1631 | 1.1627 | 1.1622 | 1.1616 | 1.1609 | 1.1601 | 1.1591 | 1.1581 |
| | 1.50 | 1.1464 | 1.1464 | 1.1462 | 1.1460 | 1.1456 | 1.1451 | 1.1445 | 1.1438 | 1.1430 | 1.1421 | 1.1410 |
| | 1.60 | 1.1305 | 1.1305 | 1.1303 | 1.1300 | 1.1297 | 1.1292 | 1.1286 | 1.1279 | 1.1271 | 1.1261 | 1.1251 |
| | 1.70 | 1.1156 | 1.1156 | 1.1154 | 1.1152 | 1.1148 | 1.1143 | 1.1137 | 1.1130 | 1.1122 | 1.1113 | 1.1103 |
| | 1.80 | 1.1017 | 1.1016 | 1.1015 | 1.1012 | 1.1008 | 1.1003 | 1.0998 | 1.0991 | 1.0983 | 1.0974 | 1.0963 |
| 1.90 | 1.0886 | 1.0885 | 1.0884 | 1.0881 | 1.0877 | 1.0872 | 1.0866 | 1.0860 | 1.0852 | 1.0843 | 1.0833 | |
| 2.00 | 1.0762 | 1.0761 | 1.0760 | 1.0757 | 1.0753 | 1.0749 | 1.0743 | 1.0736 | 1.0728 | 1.0719 | 1.0709 | |

TABLE I: $\alpha_1(\tilde{m}, \tilde{r})$: table of eigenvalues of mode 1 for $0 \leq \tilde{m} \leq 2$ and $0 \leq \tilde{r} \leq 0.1$.

*Ludovic.Bellon@ens-lyon.fr

| | | Mode 2: $\alpha_2(\tilde{m}, \tilde{r})$ | | | | | | | | | | |
|-------------|------|--|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| | | \tilde{r} | | | | | | | | | | |
| | | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | 0.10 |
| \tilde{m} | 0.00 | 4.6941 | 4.6941 | 4.6941 | 4.6941 | 4.6941 | 4.6941 | 4.6941 | 4.6941 | 4.6941 | 4.6941 | 4.6941 |
| | 0.10 | 4.3995 | 4.3987 | 4.3964 | 4.3925 | 4.3871 | 4.3801 | 4.3715 | 4.3614 | 4.3497 | 4.3363 | 4.3215 |
| | 0.20 | 4.2671 | 4.2658 | 4.2620 | 4.2556 | 4.2467 | 4.2352 | 4.2211 | 4.2044 | 4.1852 | 4.1635 | 4.1393 |
| | 0.30 | 4.1923 | 4.1906 | 4.1856 | 4.1772 | 4.1653 | 4.1501 | 4.1315 | 4.1095 | 4.0842 | 4.0557 | 4.0241 |
| | 0.40 | 4.1444 | 4.1424 | 4.1362 | 4.1259 | 4.1114 | 4.0928 | 4.0701 | 4.0433 | 4.0125 | 3.9779 | 3.9398 |
| | 0.50 | 4.1111 | 4.1087 | 4.1015 | 4.0894 | 4.0724 | 4.0506 | 4.0239 | 3.9925 | 3.9565 | 3.9163 | 3.8722 |
| | 0.60 | 4.0867 | 4.0839 | 4.0756 | 4.0618 | 4.0424 | 4.0174 | 3.9869 | 3.9511 | 3.9102 | 3.8646 | 3.8150 |
| | 0.70 | 4.0679 | 4.0648 | 4.0555 | 4.0400 | 4.0182 | 3.9901 | 3.9559 | 3.9157 | 3.8701 | 3.8194 | 3.7647 |
| | 0.80 | 4.0531 | 4.0497 | 4.0393 | 4.0221 | 3.9980 | 3.9669 | 3.9290 | 3.8846 | 3.8343 | 3.7788 | 3.7192 |
| | 0.90 | 4.0411 | 4.0373 | 4.0260 | 4.0071 | 3.9806 | 3.9465 | 3.9049 | 3.8564 | 3.8016 | 3.7416 | 3.6775 |
| | 1.00 | 4.0311 | 4.0271 | 4.0148 | 3.9942 | 3.9653 | 3.9282 | 3.8831 | 3.8305 | 3.7714 | 3.7070 | 3.6386 |
| | 1.10 | 4.0228 | 4.0184 | 4.0051 | 3.9829 | 3.9517 | 3.9116 | 3.8629 | 3.8064 | 3.7431 | 3.6745 | 3.6022 |
| | 1.20 | 4.0157 | 4.0109 | 3.9967 | 3.9728 | 3.9393 | 3.8962 | 3.8440 | 3.7836 | 3.7162 | 3.6437 | 3.5677 |
| | 1.30 | 4.0096 | 4.0045 | 3.9892 | 3.9637 | 3.9278 | 3.8818 | 3.8262 | 3.7619 | 3.6907 | 3.6144 | 3.5350 |
| | 1.40 | 4.0042 | 3.9988 | 3.9826 | 3.9554 | 3.9172 | 3.8683 | 3.8092 | 3.7412 | 3.6662 | 3.5863 | 3.5038 |
| | 1.50 | 3.9995 | 3.9938 | 3.9766 | 3.9477 | 3.9073 | 3.8554 | 3.7929 | 3.7213 | 3.6427 | 3.5594 | 3.4740 |
| | 1.60 | 3.9954 | 3.9893 | 3.9711 | 3.9406 | 3.8978 | 3.8430 | 3.7772 | 3.7021 | 3.6200 | 3.5335 | 3.4454 |
| | 1.70 | 3.9916 | 3.9853 | 3.9661 | 3.9340 | 3.8889 | 3.8312 | 3.7620 | 3.6834 | 3.5980 | 3.5086 | 3.4179 |
| | 1.80 | 3.9883 | 3.9816 | 3.9615 | 3.9277 | 3.8803 | 3.8197 | 3.7473 | 3.6653 | 3.5767 | 3.4845 | 3.3915 |
| | 1.90 | 3.9853 | 3.9783 | 3.9572 | 3.9217 | 3.8720 | 3.8086 | 3.7330 | 3.6477 | 3.5560 | 3.4611 | 3.3660 |
| | 2.00 | 3.9826 | 3.9752 | 3.9531 | 3.9161 | 3.8640 | 3.7978 | 3.7190 | 3.6306 | 3.5359 | 3.4385 | 3.3414 |

TABLE II: $\alpha_2(\tilde{m}, \tilde{r})$: table of eigenvalues of mode 2 for $0 \leq \tilde{m} \leq 2$ and $0 \leq \tilde{r} \leq 0.1$.

| | | Mode 3: $\alpha_3(\tilde{m}, \tilde{r})$ | | | | | | | | | | |
|-------------|------|--|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| | | \tilde{r} | | | | | | | | | | |
| | | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | 0.10 |
| \tilde{m} | 0.00 | 7.8548 | 7.8548 | 7.8548 | 7.8548 | 7.8548 | 7.8548 | 7.8548 | 7.8548 | 7.8548 | 7.8548 | 7.8548 |
| | 0.10 | 7.4511 | 7.4477 | 7.4374 | 7.4201 | 7.3956 | 7.3635 | 7.3237 | 7.2759 | 7.2202 | 7.1568 | 7.0866 |
| | 0.20 | 7.3184 | 7.3127 | 7.2956 | 7.2666 | 7.2252 | 7.1708 | 7.1032 | 7.0230 | 6.9315 | 6.8310 | 6.7248 |
| | 0.30 | 7.2537 | 7.2460 | 7.2228 | 7.1833 | 7.1265 | 7.0518 | 6.9595 | 6.8518 | 6.7324 | 6.6066 | 6.4796 |
| | 0.40 | 7.2155 | 7.2059 | 7.1769 | 7.1271 | 7.0553 | 6.9609 | 6.8456 | 6.7139 | 6.5727 | 6.4296 | 6.2910 |
| | 0.50 | 7.1903 | 7.1789 | 7.1441 | 7.0842 | 6.9974 | 6.8837 | 6.7469 | 6.5946 | 6.4368 | 6.2826 | 6.1383 |
| | 0.60 | 7.1725 | 7.1593 | 7.1187 | 7.0485 | 6.9468 | 6.8144 | 6.6577 | 6.4882 | 6.3183 | 6.1575 | 6.0114 |
| | 0.70 | 7.1593 | 7.1442 | 7.0979 | 7.0174 | 6.9007 | 6.7501 | 6.5756 | 6.3921 | 6.2137 | 6.0496 | 5.9040 |
| | 0.80 | 7.1490 | 7.1321 | 7.0800 | 6.9892 | 6.8575 | 6.6895 | 6.4991 | 6.3045 | 6.1206 | 5.9555 | 5.8121 |
| | 0.90 | 7.1408 | 7.1221 | 7.0642 | 6.9630 | 6.8164 | 6.6319 | 6.4276 | 6.2245 | 6.0373 | 5.8728 | 5.7325 |
| | 1.00 | 7.1341 | 7.1136 | 7.0499 | 6.9381 | 6.7769 | 6.5769 | 6.3606 | 6.1510 | 5.9623 | 5.7997 | 5.6630 |
| | 1.10 | 7.1286 | 7.1063 | 7.0367 | 6.9143 | 6.7387 | 6.5242 | 6.2977 | 6.0835 | 5.8945 | 5.7345 | 5.6019 |
| | 1.20 | 7.1239 | 7.0998 | 7.0243 | 6.8913 | 6.7015 | 6.4737 | 6.2386 | 6.0212 | 5.8331 | 5.6762 | 5.5477 |
| | 1.30 | 7.1199 | 7.0940 | 7.0126 | 6.8689 | 6.6653 | 6.4251 | 6.1830 | 5.9637 | 5.7772 | 5.6237 | 5.4994 |
| | 1.40 | 7.1164 | 7.0887 | 7.0013 | 6.8470 | 6.6299 | 6.3786 | 6.1307 | 5.9105 | 5.7262 | 5.5763 | 5.4562 |
| | 1.50 | 7.1134 | 7.0838 | 6.9905 | 6.8255 | 6.5954 | 6.3339 | 6.0814 | 5.8612 | 5.6794 | 5.5333 | 5.4172 |
| | 1.60 | 7.1108 | 7.0793 | 6.9800 | 6.8043 | 6.5616 | 6.2910 | 6.0349 | 5.8153 | 5.6364 | 5.4941 | 5.3820 |
| | 1.70 | 7.1084 | 7.0751 | 6.9697 | 6.7834 | 6.5286 | 6.2498 | 5.9910 | 5.7726 | 5.5968 | 5.4583 | 5.3500 |
| | 1.80 | 7.1063 | 7.0712 | 6.9597 | 6.7627 | 6.4964 | 6.2103 | 5.9496 | 5.7328 | 5.5602 | 5.4255 | 5.3208 |
| | 1.90 | 7.1044 | 7.0675 | 6.9499 | 6.7423 | 6.4649 | 6.1723 | 5.9104 | 5.6955 | 5.5263 | 5.3953 | 5.2941 |
| | 2.00 | 7.1027 | 7.0639 | 6.9402 | 6.7221 | 6.4341 | 6.1359 | 5.8734 | 5.6607 | 5.4949 | 5.3674 | 5.2696 |

TABLE III: $\alpha_3(\tilde{m}, \tilde{r})$: table of eigenvalues of mode 3 for $0 \leq \tilde{m} \leq 2$ and $0 \leq \tilde{r} \leq 0.1$.

Mode 4: $\alpha_4(\tilde{m}, \tilde{r})$

| | | \tilde{r} | | | | | | | | | | | | | |
|-------------|---------|-------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|--------|
| | | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | 0.10 | | | |
| \tilde{m} | 0.00 | 10.9955 | 10.9955 | 10.9955 | 10.9955 | 10.9955 | 10.9955 | 10.9955 | 10.9955 | 10.9955 | 10.9955 | 10.9955 | 10.9955 | 10.9955 | |
| | 0.10 | 10.5218 | 10.5127 | 10.4849 | 10.4372 | 10.3676 | 10.2745 | 10.1581 | 10.0210 | 9.8698 | 9.7129 | 9.5591 | 9.4054 | 9.2517 | 9.0980 |
| | 0.20 | 10.4016 | 10.3862 | 10.3387 | 10.2555 | 10.1325 | 9.9701 | 9.7771 | 9.5714 | 9.3720 | 9.1921 | 9.0377 | 8.8834 | 8.7291 | 8.5748 |
| | 0.30 | 10.3480 | 10.3269 | 10.2607 | 10.1427 | 9.9681 | 9.7450 | 9.5000 | 9.2643 | 9.0586 | 8.8894 | 8.7547 | 8.6200 | 8.4853 | 8.3506 |
| | 0.40 | 10.3178 | 10.2910 | 10.2061 | 10.0527 | 9.8278 | 9.5550 | 9.2796 | 9.0376 | 8.8423 | 8.6913 | 8.5764 | 8.4615 | 8.3466 | 8.2317 |
| | 0.50 | 10.2984 | 10.2660 | 10.1620 | 9.9726 | 9.7003 | 9.3899 | 9.1010 | 8.8654 | 8.6860 | 8.5530 | 8.4547 | 8.3564 | 8.2581 | 8.1598 |
| | 0.60 | 10.2850 | 10.2469 | 10.1235 | 9.8976 | 9.5823 | 9.2460 | 8.9550 | 8.7317 | 8.5690 | 8.4519 | 8.3669 | 8.2819 | 8.1969 | 8.1119 |
| | 0.70 | 10.2751 | 10.2313 | 10.0882 | 9.8257 | 9.4728 | 9.1205 | 8.8348 | 8.6260 | 8.4788 | 8.3751 | 8.3008 | 8.2265 | 8.1522 | 8.0779 |
| | 0.80 | 10.2675 | 10.2181 | 10.0547 | 9.7563 | 9.3715 | 9.0111 | 8.7349 | 8.5409 | 8.4077 | 8.3152 | 8.2494 | 8.1836 | 8.1178 | 8.0520 |
| | 0.90 | 10.2615 | 10.2064 | 10.0225 | 9.6889 | 9.2779 | 8.9156 | 8.6511 | 8.4714 | 8.3503 | 8.2672 | 8.2083 | 8.1494 | 8.0905 | 8.0316 |
| | 1.00 | 10.2566 | 10.1958 | 9.9911 | 9.6236 | 9.1919 | 8.8321 | 8.5803 | 8.4138 | 8.3033 | 8.2280 | 8.1749 | 8.1218 | 8.0687 | 8.0156 |
| | 1.10 | 10.2526 | 10.1860 | 9.9602 | 9.5603 | 9.1128 | 8.7587 | 8.5199 | 8.3654 | 8.2641 | 8.1954 | 8.1471 | 8.0938 | 8.0405 | 7.9872 |
| | 1.20 | 10.2492 | 10.1768 | 9.9297 | 9.4992 | 9.0403 | 8.6941 | 8.4680 | 8.3243 | 8.2309 | 8.1679 | 8.1237 | 8.0744 | 8.0251 | 7.9758 |
| | 1.30 | 10.2463 | 10.1681 | 9.8994 | 9.4403 | 8.9738 | 8.6369 | 8.4229 | 8.2890 | 8.2026 | 8.1444 | 8.1037 | 8.0584 | 8.0131 | 7.9678 |
| | 1.40 | 10.2438 | 10.1597 | 9.8694 | 9.3837 | 8.9128 | 8.5861 | 8.3836 | 8.2585 | 8.1781 | 8.1242 | 8.0864 | 8.0447 | 8.0030 | 7.9613 |
| | 1.50 | 10.2417 | 10.1516 | 9.8396 | 9.3293 | 8.8568 | 8.5408 | 8.3490 | 8.2317 | 8.1568 | 8.1065 | 8.0713 | 8.0326 | 7.9939 | 7.9552 |
| | 1.60 | 10.2398 | 10.1438 | 9.8099 | 9.2773 | 8.8054 | 8.5002 | 8.3184 | 8.2082 | 8.1380 | 8.0910 | 8.0580 | 8.0213 | 7.9846 | 7.9479 |
| 1.70 | 10.2381 | 10.1361 | 9.7804 | 9.2275 | 8.7580 | 8.4636 | 8.2912 | 8.1874 | 8.1214 | 8.0772 | 8.0463 | 8.0114 | 7.9765 | 7.9416 | |
| 1.80 | 10.2366 | 10.1286 | 9.7511 | 9.1799 | 8.7145 | 8.4307 | 8.2668 | 8.1687 | 8.1066 | 8.0650 | 8.0358 | 8.0029 | 7.9700 | 7.9371 | |
| 1.90 | 10.2352 | 10.1212 | 9.7220 | 9.1345 | 8.6743 | 8.4008 | 8.2449 | 8.1521 | 8.0933 | 8.0540 | 8.0264 | 7.9945 | 7.9626 | 7.9307 | |
| 2.00 | 10.2340 | 10.1139 | 9.6931 | 9.0913 | 8.6371 | 8.3737 | 8.2251 | 8.1370 | 8.0813 | 8.0440 | 8.0179 | 7.9870 | 7.9571 | 7.9272 | |

TABLE IV: $\alpha_4(\tilde{m}, \tilde{r})$: table of eigenvalues of mode 4 for $0 \leq \tilde{m} \leq 2$ and $0 \leq \tilde{r} \leq 0.1$.

Mode 5: $\alpha_5(\tilde{m}, \tilde{r})$

| | | \tilde{r} | | | | | | | | | | | | | |
|-------------|---------|-------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| | | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | 0.10 | | | |
| \tilde{m} | 0.00 | 14.1372 | 14.1372 | 14.1372 | 14.1372 | 14.1372 | 14.1372 | 14.1372 | 14.1372 | 14.1372 | 14.1372 | 14.1372 | 14.1372 | 14.1372 | 14.1372 |
| | 0.10 | 13.6142 | 13.5953 | 13.5364 | 13.4314 | 13.2743 | 13.0670 | 12.8284 | 12.5890 | 12.3745 | 12.1966 | 12.0553 | 11.9140 | 11.7727 | 11.6314 |
| | 0.20 | 13.5067 | 13.4742 | 13.3700 | 13.1781 | 12.8973 | 12.5727 | 12.2730 | 12.0358 | 11.8619 | 11.7373 | 11.6475 | 11.5577 | 11.4679 | 11.3781 |
| | 0.30 | 13.4615 | 13.4160 | 13.2659 | 12.9859 | 12.6053 | 12.2337 | 11.9480 | 11.7515 | 11.6196 | 11.5296 | 11.4665 | 11.4034 | 11.3403 | 11.2772 |
| | 0.40 | 13.4367 | 13.3782 | 13.1803 | 12.8138 | 12.3660 | 11.9938 | 11.7428 | 11.5833 | 11.4805 | 11.4116 | 11.3637 | 11.3158 | 11.2679 | 11.2200 |
| | 0.50 | 13.4210 | 13.3494 | 13.1019 | 12.6552 | 12.1717 | 11.8212 | 11.6053 | 11.4741 | 11.3911 | 11.3359 | 11.2975 | 11.2591 | 11.2207 | 11.1823 |
| | 0.60 | 13.4102 | 13.3253 | 13.0267 | 12.5097 | 12.0149 | 11.6941 | 11.5082 | 11.3981 | 11.3291 | 11.2833 | 11.2514 | 11.2195 | 11.1876 | 11.1557 |
| | 0.70 | 13.4023 | 13.3040 | 12.9532 | 12.3778 | 11.8882 | 11.5980 | 11.4367 | 11.3425 | 11.2837 | 11.2447 | 11.2175 | 11.1903 | 11.1631 | 11.1359 |
| | 0.80 | 13.3963 | 13.2843 | 12.8808 | 12.2594 | 11.7854 | 11.5236 | 11.3821 | 11.3002 | 11.2491 | 11.2151 | 11.1914 | 11.1677 | 11.1440 | 11.1203 |
| | 0.90 | 13.3916 | 13.2658 | 12.8096 | 12.1539 | 11.7012 | 11.4646 | 11.3392 | 11.2669 | 11.2218 | 11.1918 | 11.1709 | 11.1500 | 11.1291 | 11.1082 |
| | 1.00 | 13.3878 | 13.2479 | 12.7398 | 12.0603 | 11.6314 | 11.4169 | 11.3047 | 11.2401 | 11.1998 | 11.1730 | 11.1542 | 11.1354 | 11.1166 | 11.0978 |
| | 1.10 | 13.3846 | 13.2305 | 12.6715 | 11.9774 | 11.5730 | 11.3777 | 11.2764 | 11.2181 | 11.1817 | 11.1574 | 11.1404 | 11.1234 | 11.1064 | 11.0894 |
| | 1.20 | 13.3820 | 13.2134 | 12.6051 | 11.9040 | 11.5237 | 11.3449 | 11.2527 | 11.1997 | 11.1665 | 11.1444 | 11.1288 | 11.1132 | 11.0976 | 11.0820 |
| | 1.30 | 13.3797 | 13.1966 | 12.5408 | 11.8390 | 11.4816 | 11.3172 | 11.2327 | 11.1841 | 11.1536 | 11.1333 | 11.1190 | 11.1046 | 11.0902 | 11.0758 |
| | 1.40 | 13.3778 | 13.1798 | 12.4788 | 11.7811 | 11.4454 | 11.2934 | 11.2155 | 11.1707 | 11.1425 | 11.1237 | 11.1104 | 11.0970 | 11.0836 | 11.0702 |
| | 1.50 | 13.3761 | 13.1632 | 12.4193 | 11.7296 | 11.4139 | 11.2728 | 11.2007 | 11.1590 | 11.1329 | 11.1154 | 11.1030 | 11.0906 | 11.0782 | 11.0658 |
| | 1.60 | 13.3746 | 13.1465 | 12.3622 | 11.6836 | 11.3863 | 11.2549 | 11.1876 | 11.1489 | 11.1244 | 11.1081 | 11.0965 | 11.0849 | 11.0733 | 11.0617 |
| 1.70 | 13.3733 | 13.1299 | 12.3078 | 11.6424 | 11.3620 | 11.2391 | 11.1762 | 11.1398 | 11.1170 | 11.1016 | 11.0907 | 11.0791 | 11.0675 | 11.0559 | |
| 1.80 | 13.3721 | 13.1132 | 12.2558 | 11.6052 | 11.3404 | 11.2251 | 11.1660 | 11.1318 | 11.1103 | 11.0958 | 11.0856 | 11.0749 | 11.0642 | 11.0535 | |
| 1.90 | 13.3711 | 13.0966 | 12.2065 | 11.5718 | 11.3212 | 11.2125 | 11.1569 | 11.1247 | 11.1043 | 11.0906 | 11.0810 | 11.0703 | 11.0606 | 11.0509 | |
| 2.00 | 13.3701 | 13.0798 | 12.1596 | 11.5414 | 11.3039 | 11.2013 | 11.1487 | 11.1182 | 11.0989 | 11.0860 | 11.0768 | 11.0676 | 11.0584 | 11.0492 | |

TABLE V: $\alpha_5(\tilde{m}, \tilde{r})$: table of eigenvalues of mode 5 for $0 \leq \tilde{m} \leq 2$ and $0 \leq \tilde{r} \leq 0.1$.