LIP Research Report RR2006-51

CP games: a tutorial

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December 20, 2006

Abstract

In this tutorial we introduce the concept of CP-game which is a generalization of this of strategic game. First we present examples which are relevant to a CP-game approach. Then we give a somewhat naive introduction to CP-games. Then we present the connection between CP games and gene regulation networks as presented by René Thomas.

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1 Introduction

This paper aims to be a didactic introduction to concepts developed in [3] and used in [1] as a formalization of gene regulation networks. CP games rely on *players* and *synopses* that are alos game situations. A player may change a synopsis to another provided it can be converted to, moreover a synopsis can be preferred to another. *Conversion* and *preference* are two basic concepts of CP games from which they take their name (C and P). The following tutorial presents all those concepts through examples.

2 Basic concepts

To start with, let us give the two main notions of games. First, with no surprise, a game involves *players*. Second, a game is characterized by situations. In CP games these situations will be called *synopses* or sometimes *game situations*. A player can move from one synopsis to another, but she[1] does that under some constraints as she has no total freedom to perform her moves, therefore a relation called *conversion* is defined for each player; it tells what moves a player is allowed to perform. Conversion of player *Alice* will be written $\blacktriangleright_{Alice}$. As such, conversion tells basically the rules of the game. In chess it would say *"a player can move her bishop along a diagonal"*, but it does not tell the game line of the player. In other words it does not tell why the player chooses to move or how to convert her synopsis. Another relation called *preference* compares synopses in order for a player to choose a better move or to perform a better conversion. Preference of player *Beth* will be written $\triangleright_{Beth}$ and when we write $s \triangleright_{Beth} s'$ we mean that *Beth* prefers s' to s or rather than s she chooses s'. Preference (or choice) is somewhat disconnected from conversion, a player can clearly prefer a synopsis she cannot move to and vice versa she can move to a synopsis she does not prefer.

A key concept in games is this of *equilibrium*. As a player can convert a synopsis, she can convert it to a synopsis she likes better, in the sense that she prefers the new synopsis she converted to. A player is *happy* in a synopsis, if there is no synopsis she can convert to and she prefers. A synopsis is an equilibrium if each player is happy with this synopsis. We will see that this concept of equilibrium captures and generalizes the concepts known as *Nash equilibrium* in strategic games .

3 Some examples

Let us present the above concepts of conversion, preference and equilibrium through examples. We will introduce a new concept called *change of mind*.

3.1 A simple game on a square

As an introduction, we will look at variations of a simple game on a board.

[1] See the preface of [2] for the use of personal pronouns

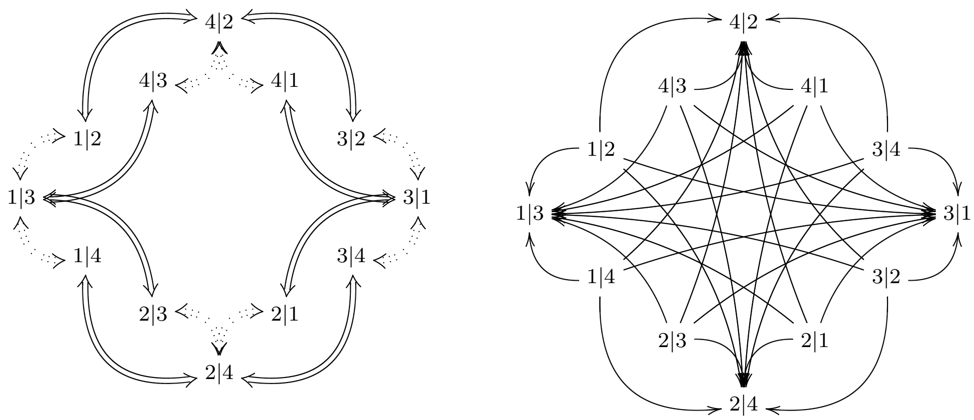
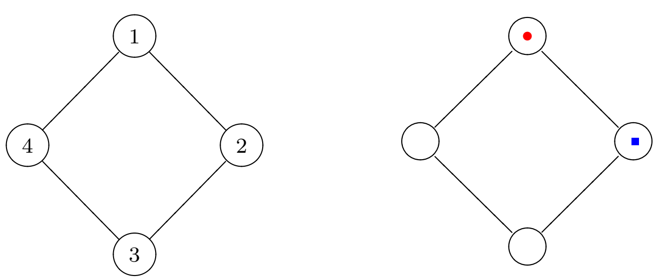


Figure 1: Conversion and preference for the square game

3.1.1 A first version

Imagine a simple game where *Alice* and *Beth* play using tokens on a square. We number the four positions as 1, 2, 3 and 4.



Assume that player *Alice* has a red round token and that player *Beth* has a blue squared token. The two player places their tokens on one vertices and then they move along edges. They can also decide not to move. Assume that *Alice* and *Beth* never put their token on a vertex taken by the other player, more precisely a move towards an occupied vertex corresponds to a capture of the token at that vertex and to a win. Then the game ends.

The game has 12 situations or synopses, which we write $i|j$ for $1 \leq i, j \leq 4$ and $i \neq j$. The above pictured situation corresponds to 1|2. The two conversions are described by Figure 1 left. In this figure, $\Longrightarrow$ is *Alice*'s conversion and $\dashrightarrow$ is *Beth*'s conversion.

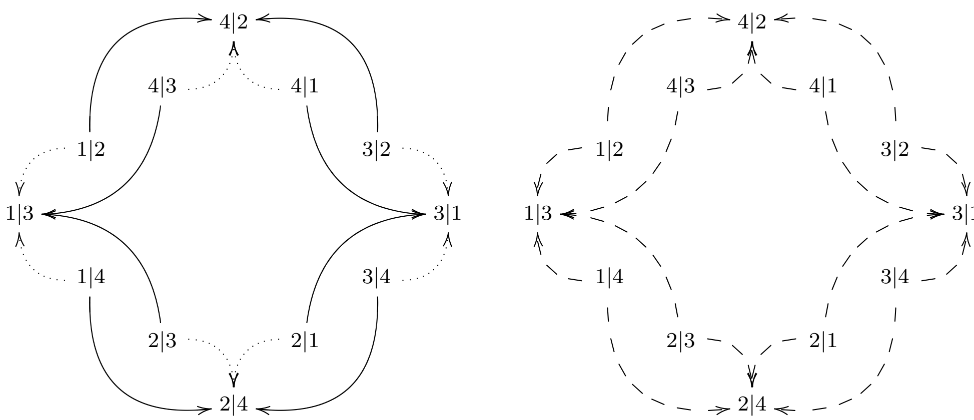


Figure 2: Agent changes of mind (on the left) and (general) change of mind (on the right) for the square game

In this game, both players share the same preference, namely the following: since a player does not want her token to be captured, she prefers a synopsis where her token iis on the opposite corner of the other token to a synopsis where her token is next to the other token. This gives the preference given in Figure 1 right. The arrow from 1|2 to 1|3 means players prefer 1|3 to 1|2.

From the conversion and the preference we build a relation that we call *change of mind*. *Alice* can change her mind from a synopsis s to a new one s', if she can convert s to the new synopsis s' and rtaher than s she chooses s'. Changes of mind for *Alice* and *Beth* are given in Figure 2 on the left. In this figure, $\longrightarrow$ is *Alice*'s change of mind and ⋯⋯> is *Beth*'s conversion. The *(general) change of mind* is the union of the *agent change of mind*, it is given by Figure 2 on the right. The equilibria are the sinks for that relation, namely 1|3, 4|2, 3|1 and 2|4. This means that no change of mind arrows leave those nodes. In these situations players have their tokens on opposite corners and they do not move. An equilibrium like 1|3 which is a sink is called a *Abstract Nash Equilibrium*.

3.1.2 A second version

We propose a second version of the game, where moves of the token can only be made clockwise. Obviously the conversion changes, but also the preference, as a player does want not be threatened by another token placed before her clockwise and prefers a synopsis that places this token as far as possible. The conversions, the preferences and the changes of mind are given in Figure 8. If one looks at the equilibrium, one sees that there is no fixed position where players are happy. To be happy the players have to move around for ever, one chasing the other. It is not really a cycle, but a perpetual move. We also call that an equilibrium. It is sometime called a *dynamic equilibrium* or a *stationary state* according to people you talk to.

3.1.3 A third version

The third version is meant to present an interesting feature of the change of mind. In this version, we use the same rules as the second one, except that we suppose that the game starts with both token on the board or it starts as follows. *Alice* has put her token on node 1 (this game positions is described as $1|\omega$). Then *Beth* chooses a position among 2, 3 or 4. The conversion is given in Figure 3 left. *Beth* may choose not to play, but in this case she looses, in other words, she prefers[2] any position to $1|\omega$. The change of mind is given on Figure 3 right. There is again a dynamic equilibrium and one sees that this dynamic equilibrium is not the whole game, indeed one enters the perpetual move after at least one step in the game.

3.2 Strategic games

In this presentation of strategic games we do not use payoff functions, but directly a preference relation[3] and we present several games.

3.2.1 The Prisoner's Dilemma

The problem is stated usually as follows

> Two suspects, A and B, are arrested by the police. The police have insufficient evidence for a conviction, and, having separated

[2]We do not draw the preference that would become to entangled.

[3]See Section 1.1.2 of [2] for a discussion.

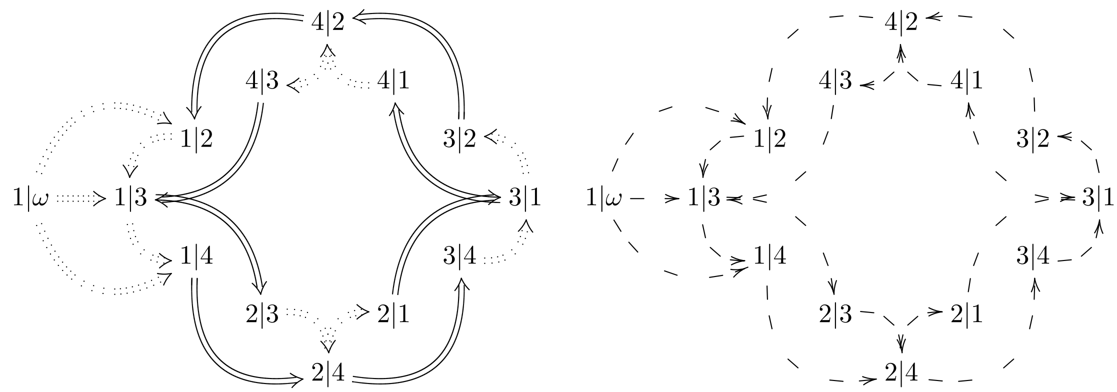


Figure 3: Conversion and change of mind for third version of the square game

> both prisoners, visit each of them to offer the same deal: if one acts as an informer against the other (*finks*) and the other remains *quiet*, the betrayer goes free and the quiet accomplice receives the full sentence. If both stay quiet, the police can sentence both prisoners to a reduced sentence in jail for a minor charge. If each finks, each will receive a similar intermediate sentence. Each prisoner must make the choice of whether to fink or to remain quiet. However, neither prisoner knows for sure what choice the other prisoner will make. So the question this dilemma poses is: What will happen? How will the prisoners act?

Each prisoner can be into two states, either *fink* (F) or be *quiet* (Q). Each prisoner can go from Q to F and vice-versa, hence the following conversion, where $\Longrightarrow$ is prisoner A conversion and $\cdots\!>$ is prisoner B conversion (Figure 4 left). Each prisoner prefers to go free to been sentenced and prefers a light sentence to a full sentence. Hence the preference are given in Figure 4 right without the arrows than can be deduced by transitivity, where $\longrightarrow$ is prisoner A preference and $\cdots\!>$ is prisoner B preference.

From this we get the change of mind of Figure 5. One sees clearly that the only equilibrium is F, F despite both prefer Q, Q as shown on Figure 4 right.

Such an equilibrium is called a Nash equilibrium in strategic game theory.

Figure 4: Conversion and preference in the prisoner's dilemma

Figure 5: Agent and (general) change of mind in the prisoner's dilemma

3.2.2 Matching Pennies

This second example is also classic. This is a simple example of strategic game where there is no singleton equilibrium.

> The game is played between two players, Player A and Player B. Each player has a penny and must secretly turn the penny to heads (H) or tails (T). The players then reveal their choices simultaneously. If the pennies match (both heads or both tails), Player A wins. If the pennies do not match (one heads and one tails), Player B wins.

The conversion is similar to this of the prisoner's dilemma (Figure 6 left) and the preference is given by who wins (Figure 6 center).

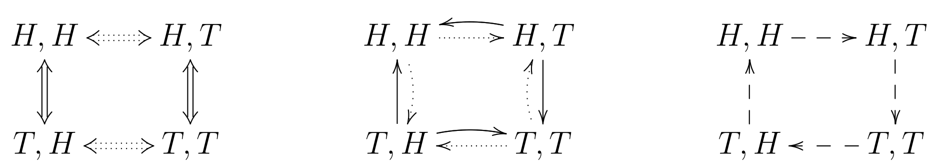
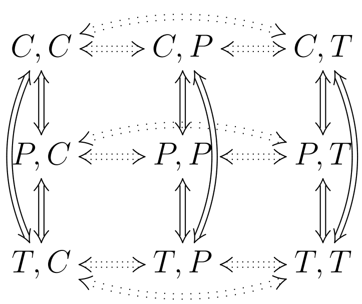


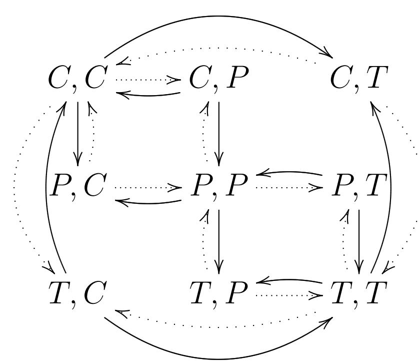
Figure 6: Conversion, preference and (general) change of mind in Matching Pennies

Change of mind for matching pennies is in Figure 6 right. One notices that there is cycle. This cycle is the equilibrium. No player has clear mind of what to play and changes her minds each time she looses.

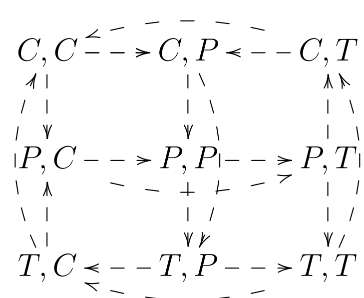
3.2.3 Scissors, paper, stone

Here we present the famous game known as *scissors, paper, stone*. It involves two players, *Alice* and *Beth* who announce either *scissors* (C) or *paper* (P) or *stone* (T) with the rules that *stone beats scissors, scissors beat paper, and paper beats stone*. There are nine situations (see below), one sees that *Alice* may convert her situation P, C to P, P or P, T and the same for the other situations. The conversion is given below left. Since the rules, it seems clear that *Alice* prefers T, P to P, P and P, P to C, P, hence the preference given below right with $\longrightarrow$ is *Alice*'s preference and $\dashrightarrow$ is *Beth*'s preference. To avoid a cumbersome diagram, in the preference we do not put the arrows deduced by transitivity.





From the above conversion and preference, one gets the following change of mind.



One sees also perpetual moves as in the matching pennies of which it is a generalization.

3.2.4 Strategic games as CP games

A strategic game is a specific kind of CP games. To be a strategic game, a CP game has to fulfill the following conditions.

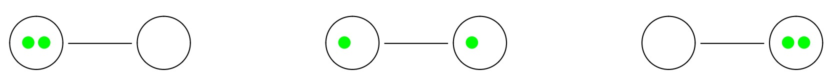
1. Each synopsis is a n-Cartesian product, where n is the number of players. The constituents of the Cartesian product are called *strategies*.

2. Conversion for player a, written $\blacktriangleright_a$, is any change along the a-th dimension, i.e., $(s_1, ..., s_a, ..., s_n) \blacktriangleright_a (s_1, ..., s'_a, ..., s_n)$ if and only if $s_a \neq s'_a$. Hence in strategic games, conversion is

   - symmetric, ($s \blacktriangleright_a s'$ implies $s' \blacktriangleright_a s$),
   - transitive, ($s \blacktriangleright_a s'$ and $s' \blacktriangleright_a s''$ imply $s \blacktriangleright_a s''$),
   - and irreflexive (it is not true that $s \blacktriangleright_a s$).

3.3 Blink or you loose

Blink and you loose is a game played on a simple graph with two undifferentiated tokens. There are three positions:



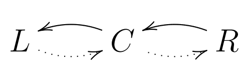
There are two players, *Left* and *Right*. The leftmost position above is the winning position for *Left* and the rightmost position is the winning position for *Right*. In other words, the one who owns both token is the winner. Let us call the positions L, C and R respectively. One plays by taking a token on the opposite node.

3.3.1 A first tactic: Foresight

A player realizes that she can win by taking the opponent's token faster than the opponent can react, i.e., player *Left* can convert C to L by outpacing player *Right*. Player *Right*, in turn, can convert C to R. This version of the game has two singleton equilibria: L and R. This is described by the following conversion

$$L \Longleftarrow C \dashrightarrow R$$

where $\Longrightarrow$ is the conversion for *Left* and $\dashrightarrow$ is the conversion for *Right*. The preference is



where $\longrightarrow$ is the preference for *Left* and $\dashrightarrow$ is the preference for *Right*. The change of mind is then:

$$L \dashleftarrow C \dashrightarrow R$$

and one sees that there are two equilibria: namely L and R, which means that players have taken both token and keep them.

3.3.2 A second tactic: Hindsight

A player, say *Left*, analysis what would happen if she does not act. In case *Right* acts, the game would end up in R and *Left* looses. As we all know, people hate to loose so they have an aversion for a loosing position. Actually *Left* concludes that she could have prevented the R outcome by acting. In other words, it is within *Left*'s power to convert R to C. Similarly for player *Right* from L to C.

$$L \dashrightarrow C \Longleftarrow R$$

We call naturally *aversion* the relation that escapes from positions a player does not want to be, especially a loosing position. Aversion deserves its name as it works like conversion, but flies from bad position. We get the following change of mind:

$$L \dashrightarrow C \dashleftarrow R$$

with C the singleton equilibrium

3.3.3 A third tactic: Omnisight

The players have both hindsight and foresight, resulting in a C/P game

$$L \underset{\dashrightarrow}{\overset{\Longleftarrow}{}} C \underset{\dashrightarrow}{\overset{\Longleftarrow}{}} R$$

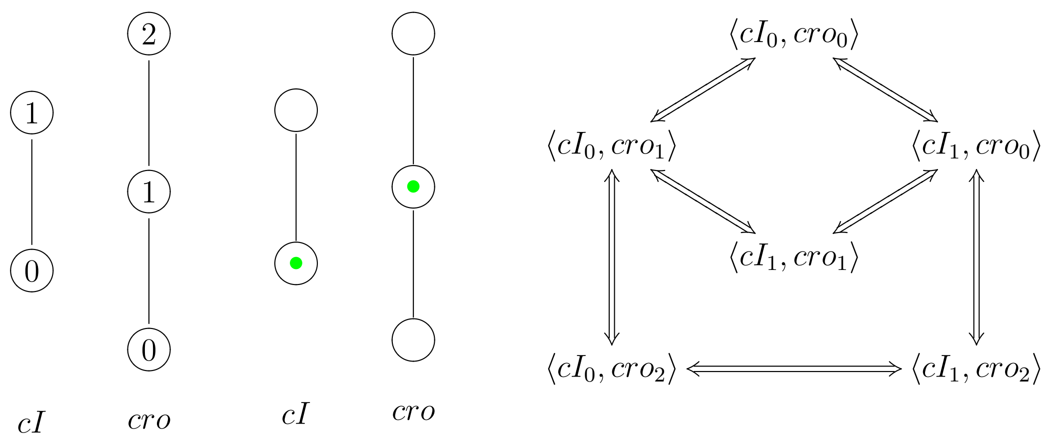
with one change-of-mind equilibrium covering all outcomes thus, no singleton equilibrium equilibrium exists.

$$L \underset{\dashleftarrow}{\overset{\dashrightarrow}{}} C \underset{\dashleftarrow}{\overset{\dashrightarrow}{}} R$$

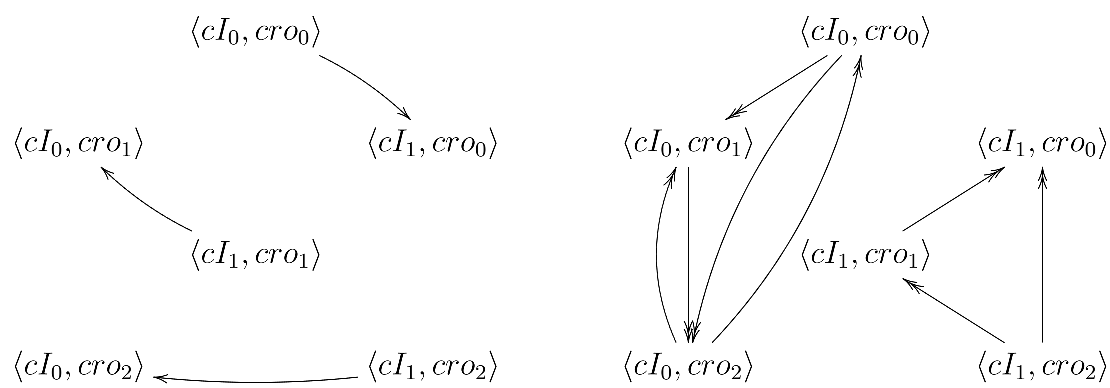
3.4 The λ phage as a CP game

The λ phage is a game inspired from biology [1]. The origin of the game will be given in Section 6, now we will give just *the rules of the game.*

There are two players cI and cro and the game can be seen as a game with two tokens moving on two graphs where each player may choose to move one of the two tokens[4]. The conversion is therefore the same for both players[5] and is given by the following rightmost diagram:



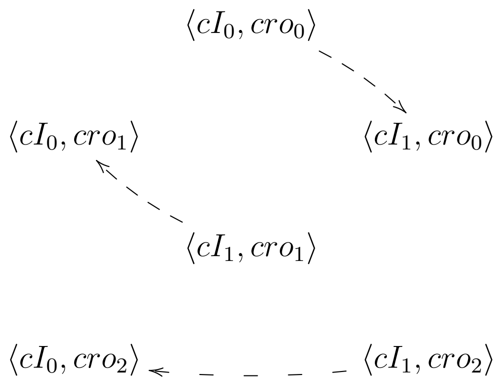
The preference is difficult to describe as an actual game to be played, it comes from the genetics and is specific to each player. Being a gene, I prefer a position if I am "pushed forward" that position.

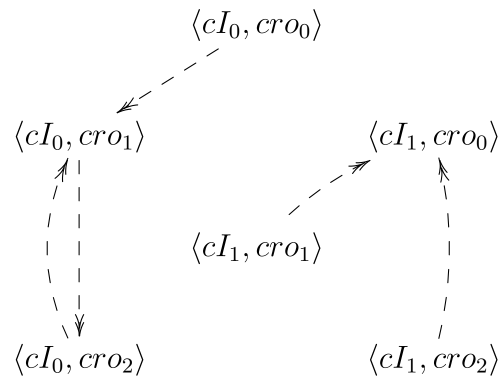


[4] In the asynchronous version.

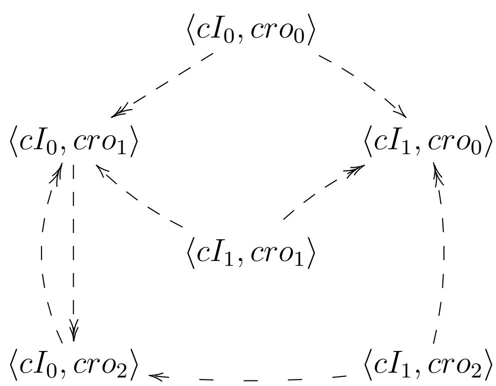
[5] Note the difference with the square game where players had different conversions and the same preference. The fact that the conversion is the same for everybody seems to be a feature of biologic game.

From the conversion and the preferences one deduces two changes of mind.





from which we deduce the (general) change of mind of the game:



One sees one singleton equilibrium namely $\langle cI_1, cro_0\rangle$ and one dynamic equilibrium namely $\{\langle cI_0, cro_1\rangle, \langle cI_0, cro_2\rangle\}$.

4 Presentation of C/P games

To define a C/P game we have to define four concepts:

- a set $\mathcal{A}$ of *agents*,
- a set $\mathcal{S}$ of *synopses*,
- for every agent a a relation $\blacktriangleright_a$ on $\mathcal{S}$, called *conversion*,
- for every agent a a relation $\triangleright_a$ on $\mathcal{S}$, called *preference*.

From these relations we are going to define a relation called *change of mind*.

Definition 1 (Game) *A game is a 4-uple* $\langle \mathcal{A}, \mathcal{S}, (\blacktriangleright_a)_{a\in\mathcal{A}}, (\triangleright_a)_{a\in\mathcal{A}}\rangle$.

Example 1 (Square game 1rst version) *For the first version of the square game we have:*

- $\mathcal{A} = \{Alice, Beth\}$,
- $\mathcal{S} = \{1|2, 1|3, 1|4, 2|3, 2|4, 2|1, 3|4, 3|1, 3|2, 4|1, 4|2, 4|3\}$,
- *Conversions* $\blacktriangleright_{Alice}$ *and* $\blacktriangleright_{Beth}$ *are given by Figure 1 left, with* $\Longrightarrow$ *for* $\blacktriangleright_{Alice}$ *and* $\dashrightarrow$ *for* $\blacktriangleright_{Beth}$,
- $\triangleright_{Alice} = \triangleright_{Beth}$ *and this relation is given by Figure 1 right.*

4.1 Singleton equilibrium

Let us look at a first kind of equilibria.

Definition 2 (Singleton equilibrium) *A* singleton equilibrium *is a synopsis* s *such that:*

$$\forall a \in \mathcal{A}, s' \in \mathcal{S} \quad . \quad s \blacktriangleright_a s' \Rightarrow \neg(s \triangleright_a s').$$

In the previous paragraphs, we have seen examples of singleton equilibria. If we are at such an equilibrium, this is fine, but if not, we may wonder how to reach an equilibrium. If s is not an equilibrium, this means that s fulfills

$$\exists s' \in \mathcal{S} \quad . \quad s \blacktriangleright_a s' \wedge s \triangleright_a s'$$

which is the negation of

$$\forall s' \in \mathcal{S} \quad . \quad s \blacktriangleright_a s' \Rightarrow \neg(s \triangleright_a s').$$

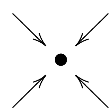
The relation $s \blacktriangleright_a s' \wedge s \triangleright_a s'$ between s and s' is new. Let us call it *change of mind for* a and write it $\to_a$. We say that a changes her mind, because she is not happy with s and hopes that following $\to_a$ she will reach not necessary the equilibrium, but at least a better situation. Actually since we want to make everyone happy, we have to progress along all the $\to_a$'s. Thus we consider a more general relation which we call just *change of mind* and which is the union of the $\rightarrow_a$'s, i.e.,

$$\rightarrow \triangleq \bigcup_{a \in \mathcal{A}} \rightarrow_a .$$

Now suppose that we progress along $\rightarrow$. What happens if we reach an s from which we cannot progress further? This means

$$\forall a \in \mathcal{A}, s' \in \mathcal{S} \quad . \quad \neg(s \blacktriangleright_a s' \wedge s \triangleright_a s')$$

in other words, s is an equilibrium. Hence to reach an equilibrium, we progress along $\rightarrow$ until we are stuck. In graph theory, a vertex from which there is no outgoing arrow is called a *sink*.



Thus we look for sinks in the graph.

4.2 Dynamic equilibrium

Actually this progression along $\rightarrow$ is not the panacea to reach an equilibrium. Indeed it could be the case that this progression never ends, since we enter a perpetual move (think at the square game 2nd version, Figure 3). Actually we identify this perpetual move as a second kind of equilibrium.

4.2.1 Strongly connected components

Here it is relevant to give some concepts of graph theory. A graph[6] is *strongly connected*, if given two nodes n_1 and n_2 there is always a path going from n_1 to n_2 and a path going from n_2 to n_1. Obviously not all the graphs are strongly connected, but they may contain some maximal subgraphs that are strongly connected; "maximal" means that one cannot add nodes without breaking the strong connectedness. Such a strongly connected subgraph is called a *strongly connected components*, *SCC* in short.

[6]In this tutorial, when we say "graph" , we mean always "oriented graph".

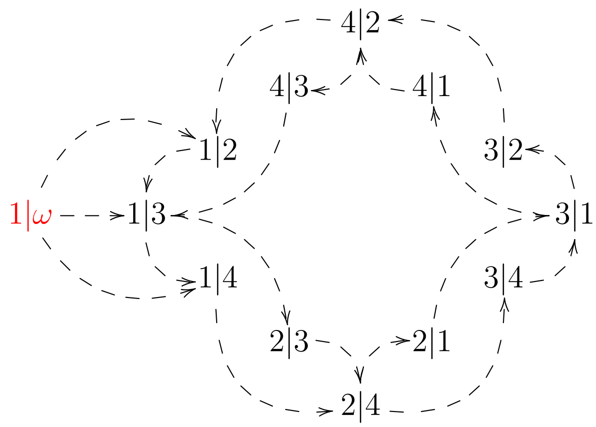
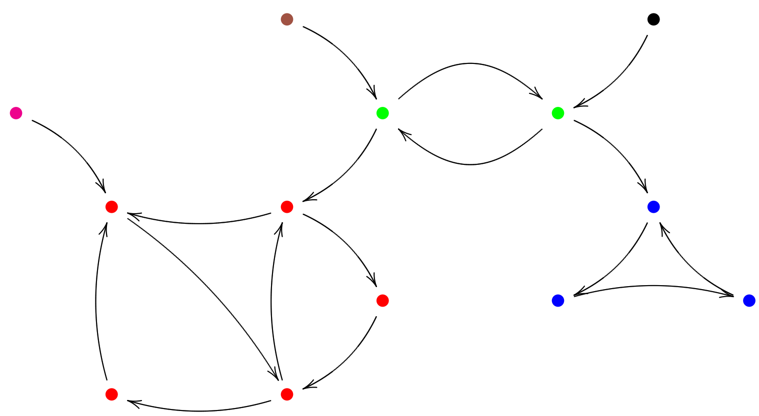


Figure 7: A game with two SCC's

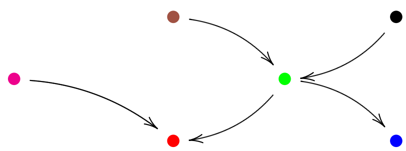
The graph below has six SCC's:



The graph of Figure 3 has two SCC's (Figure 7)

From a graph, we can deduce a new graph, which we call the *reduced graph*, whose nodes are the SCC's and the arcs are given as follows: there is an arc from an SCC S_1 to an SCC S_2, if there exists a node n_1 in S_1, a node s_2 in S_2 and an arc between n_1 and n_2. By construction the reduced graph has no cycle and its strongly connected components are singletons.

The reduced graphs associated with the graphs given above are as follows:



$$\{1|\omega\} \dashrightarrow \{1|3, 1|2, 1|4, ...\}$$

4.2.2 Dynamic equilibria as strongly connected components

At the price of extending the notion of equilibrium, we can prove that there is always an equilibrium. Indeed given a graph, we compute its reduced graph. Then in this reduced graph, we look for sinks. There is always such a sink since in an acyclic graph (the reduced graph is always acyclic) there exists always at least a sink.

CP Equilibria *are* sinks *in the reduced graph.*

We can now split equilibria into two categories?

1. *CP Equilibria* (i.e., *Dynamic equilibria* are equilibria associated with an SCC that contains more that one synopsis.

2. *Abstract Nash Equilibria* (i.e., *Singleton equilibria*) are equilibria associated with an SCC that contains exactly one synopsis, i.e., associated with an SCC which is a singleton, hence the name singleton equilibrium.

5 Gene regulation networks as CP games

A state based presentation of gene regulation networks can be made following Thomas approach [5, 4]. It is asynchronous: basically one gene changes its state one level at a time. Here are the features.

Genes [7] are basic concepts. The form a set G. For instance in the following we consider, as an arbitrary example, the set $G_4 = \{ga, gb, gc, gd\}$. Each element g of G is associated with a finite non empty set: the set of "states" of the gene g or the set of "levels of activation" of the gene g,

[7] We say "genes", but it could be proteins or something completely different.

pictured by a diagram like this of λ phage in Section 3.4. If gene g has three levels they will be written g_0, g_1, g_2 and the set $\{g_0, g_1, g_2\}$ will be written S_g. Those levels are ordered by the order given by their indices e.g., $g_0 < g_1 < g_2$.

A state of the network is a n-uple where n is the number of genes. The component of a state s that corresponds to the gene g is written s_g. The set of states of the set of genes G is written S_G. In the above example G_4, a state is a quadruple; $\langle ga_1, gb_2, gc_0, gd_1\rangle$ is such a state.

A regulation is a relation between genes. When we write $ga \curvearrowright gb$ we mean that gene ga regulates gene gb; ga can activate gb or ga can inhibit gb. Let us write $\mathsf{Regulators}(g)$ the set of genes that regulate the gene g. For instance in the above example, we may have $\mathsf{Regulators}(ga) = \{ga, gb, gc\}$.

Comfort functions are functions that say more about the regulation. A comfort function, written K_g, is associated to each gene g, its domain is a state of $\mathsf{Regulators}(g)$ and its results is a state of g. For instance with gene ga, K_{ga} is a function from $S_{ga} \times S_{gb} \times S_{gc}$ to S_{ga} since $\mathsf{Regulators}(ga) = \{ga, gb, gc\}$. For instance, $K_{ga}(\langle ga_1, gb_0, gc_2\rangle) = ga_0$.

We extend the domain of comfort functions by adding dummy values. Indeed it is not really convenient to say that the comfort function K_g depends only on states in $S_{\mathsf{Regulators}(g)}$ (and only on that state), because this means that each function has its own domain. Instead we change each comfort function into a function with the domain S_G. For instance we create the function $\overline{K}_{ga}(\langle ga_1, gb_0, gc_2, gd_0\rangle)$ which takes basically the same value as K_{ga}, but to insist on the fact that the result of $\overline{K}_{ga}$ does not depend on gd (indeed $\overline{K}_{ga}(\langle ga_1, gb_0, gc_2, gd_0\rangle) = \overline{K}_{ga}(\langle ga_1, gb_0, gc_2, gd_1\rangle)$) we write $\overline{K}_{ga}(\langle ga_1, gb_0, gc_2, \_\rangle)$ and we call "_" a dummy value. This way

$$Dom(\overline{K}_{ga}) = Dom(\overline{K}_{gb}) = Dom(\overline{K}_{gc}) = Dom(\overline{K}_{gd}) = S_G.$$

A CP game can be associated to a state based presentation. Since we associate a CP game with a presentation, we must provide its agents, its synopses, its conversions and its preferences.

The agents are the genes.

The synopses are the states.

The conversion says how we may move from one state to another. Since Thomas' approach is asynchronous, this means that we can move from a network state to another by just changing the state of one gene[8]. For instance, $\langle ga_1, gb_0, gc_2, gd_0\rangle \blacktriangleright \langle ga_1, gb_0, gc_2, gd_1\rangle$. Notice that since the conversion is the same for all agents, we write $\blacktriangleright$ instead of $\blacktriangleright_g$.

The preference is dictated by comfort functions. It says that a gene g prefers a state s' to a state s if the gene state s'_g, i.e., the g-component of s', is between s_g and $\overline{K}_g(s)$, i.e., we have either $s_g < s'_g \leq \overline{K}_g(s)$ or $s_g > s'_g \geq \overline{K}_g(s)$. In the CP game notations this means

$$s \rhd_g s' \quad \text{if by definition } (s_g < s'_g \leq \overline{K}_g(s)) \vee (s_g > s'_g \geq \overline{K}_g(s)).$$

6 Back to the λ phage

The λ-phage est the game we have seen in Section 3.4. Let us see how it can be presented by the previous formalism and how we can derive a C/P game.

The agents are two: cI and cro.

The synopses are six, since cI has two states 0 and 1 and cro has three states 0, 1 and 2. Those states are therefore:

$$\langle cI_0, cro_0\rangle, \langle cI_0, cro_1\rangle, \langle cI_0, cro_2\rangle, \langle cI_1, cro_0\rangle, \langle cI_1, cro_1\rangle, \langle cI_1, cro_2\rangle.$$

The conversion corresponds to a move, one state at a time, as shown on page 12.

The preference is given by $\overline{K}(\langle cI_i, cro_i\rangle)$. According to Thomas, one has

$$\begin{array}{rcl} K_{cI}(\langle \_, cro_0\rangle) &=& cI_1 \\ K_{cI}(\langle \_, cro_1\rangle) &=& cI_0 \\ K_{cI}(\langle \_, cro_2\rangle) &=& cI_0 \end{array} \qquad \begin{array}{rcl} K_{cro}(\langle cI_0, cro_0\rangle) &=& cro_2 \\ K_{cro}(\langle cI_0, cro_1\rangle) &=& cro_2 \\ K_{cro}(\langle cI_0, cro_2\rangle) &=& cro_0 \\ K_{cro}(\langle cI_1, \_\rangle) &=& cro_0 \end{array}$$

[8]A synchronous approach would say that we can change more than one state at a time. Synchronous systems of genes can also be described by CP games.

Therefore for $\rhd_{cI}$ one gets

$$\begin{array}{lcll}
\langle cI_0, cro_0\rangle & \rhd_{cI} & \langle cI_1, cro_0\rangle & \textbf{since } cI_0 < cI_1 \leq K_{cI}(\langle _, cro_0\rangle) = cI_1. \\
\langle cI_1, cro_1\rangle & \rhd_{cI} & \langle cI_0, cro_1\rangle & \textbf{since } cI_1 > cI_0 \geq K_{cI}(\langle _, cro_1\rangle) = cI_0. \\
\langle cI_1, cro_2\rangle & \rhd_{cI} & \langle cI_0, cro_2\rangle & \textbf{since } cI_1 > cI_0 \geq K_{cI}(\langle _, cro_1\rangle) = cI_0.
\end{array}$$

and for $\rhd_{cro}$ one gets

$$\begin{array}{lcll}
\langle cI_0, cro_0\rangle & \rhd_{ro} & \langle cI_0, cro_1\rangle & \textbf{since } cro_0 < cro_1 \leq K_{cro}(\langle cI_0, cro_0\rangle) = cro_2. \\
\langle cI_0, cro_0\rangle & \rhd_{ro} & \langle cI_0, cro_2\rangle & \textbf{since } cro_0 < cro_2 \leq K_{cro}(\langle cI_0, cro_0\rangle) = cro_2. \\
\langle cI_0, cro_1\rangle & \rhd_{ro} & \langle cI_0, cro_2\rangle & \textbf{since } cro_1 < cro_2 \leq K_{cro}(\langle cI_0, cro_1\rangle) = cro_2. \\
\langle cI_0, cro_2\rangle & \rhd_{ro} & \langle cI_0, cro_1\rangle & \textbf{since } cro_2 > cro_1 \geq K_{cro}(\langle cI_0, cro_2\rangle) = cro_0. \\
\langle cI_0, cro_2\rangle & \rhd_{ro} & \langle cI_0, cro_0\rangle & \textbf{since } cro_2 > cro_0 \geq K_{cro}(\langle cI_0, cro_2\rangle) = cro_0. \\
\langle cI_1, cro_1\rangle & \rhd_{ro} & \langle cI_1, cro_0\rangle & \textbf{since } cro_1 > cro_0 \geq K_{cro}(\langle cI_1, _\rangle) = cro_0. \\
\langle cI_1, cro_2\rangle & \rhd_{ro} & \langle cI_1, cro_1\rangle & \textbf{since } cro_2 > cro_1 \geq K_{cro}(\langle cI_1, _\rangle) = cro_0. \\
\langle cI_1, cro_2\rangle & \rhd_{ro} & \langle cI_1, cro_0\rangle & \textbf{since } cro_2 > cro_0 \geq K_{cro}(\langle cI_1, _\rangle) = cro_0.
\end{array}$$

The reader may check that this corresponds to the pictures of Section 3.4.

7 Conversion or preference, how to choose?

The attentive reader may have noticed that what counts is the *change of mind* and that there is some freedom on the conversion and the preference provided one keeps the same change of mind. More precisely, we have

$$\begin{array}{lcll}
\rightarrow_a & = & \blacktriangleright_a \cap \rhd_a & \\
 & = & (\blacktriangleright_a \cup R) \cap \rhd_a & \text{where } R \cap \rhd_a = \emptyset \\
 & = & \blacktriangleright_a \cap (\rhd_a \cup T) & \text{where } T \cap \blacktriangleright_a = \emptyset
\end{array}$$

On another hand, one notices that in some examples, the preference is independent of the agent whereas in others, the conversion is independent of the agent. It seems that this is correlated with the domain of application. In particular, we may emit the following hypothesis. In biology, conversion is physics and chemistry, whereas preference is the result of evolution, then we may induce that change of mind (combination of physics and evolution) is life. Indeed since physics and chemistry is the same for everyone, it makes sense to say that conversion is the same for everybody, whereas due to evolution preference, changes with agents.

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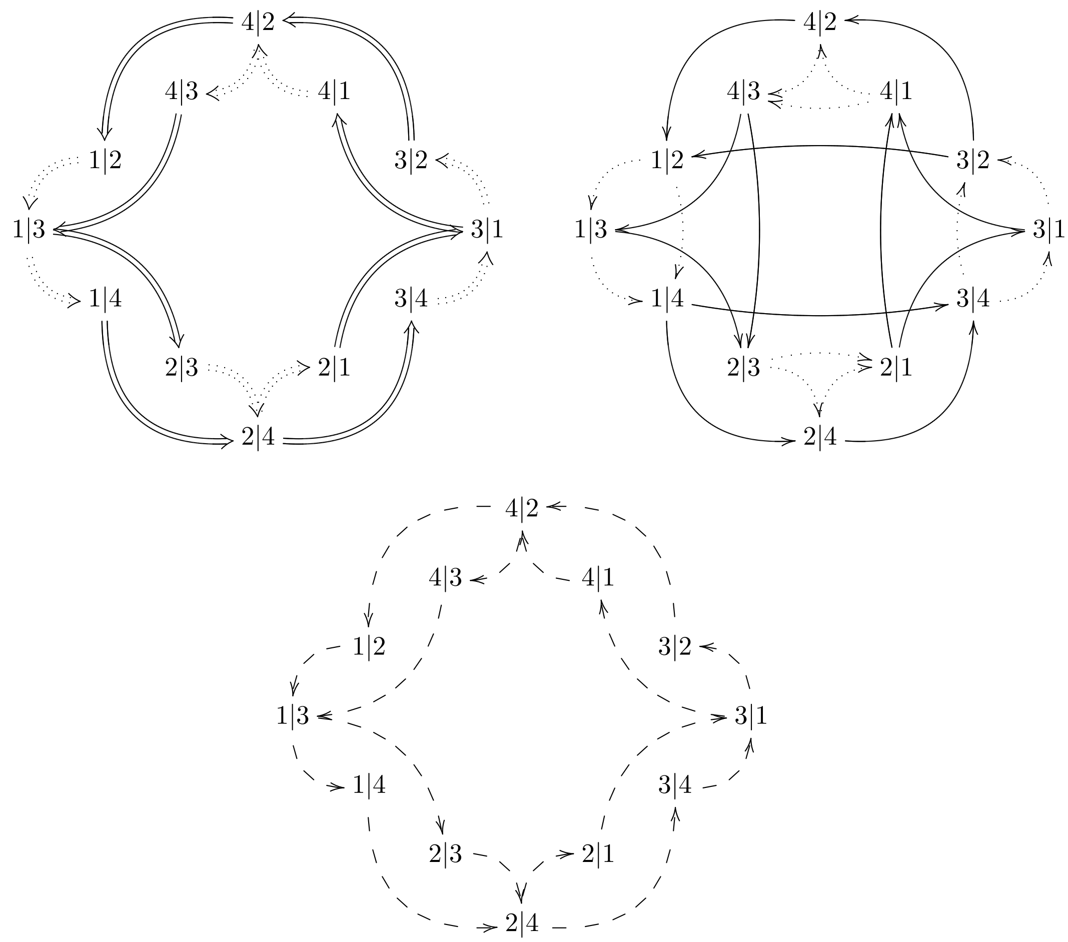

Figure 8: Conversion, preference and change of mind for the second version of the square game