Creep motion of a granular pile induced by thermal cycling

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We report a time-resolved study of the dynamics associated with the slow compaction of a granular column submitted to thermal cycles. The column height displays a complex behavior: for a large amplitude of the temperature cycles, the granular column settles continuously, experiencing a small settling at each cycle; By contrast, for small-enough amplitude, the column exhibits a discontinuous and intermittent activity: successive collapses are separated by quiescent periods whose duration is exponentially distributed. We then discuss potential mechanisms which would account for both the compaction and the transition at finite amplitude.

Literature canonically presents granular materials as a bunch of athermal particles [1–3]. Indeed, the energy necessary for the grains to hop one over another (a few tenths of millimeter, for instance) is roughly 10 orders of magnitude higher than the ambient thermal agitation k_BT. This is probably one of the reasons why the behavior of a granular assembly submitted to temperature fluctuations has received so few attentions. Nonetheless, uncontrolled thermal dilations of a granular pile have been reported to generate stress fluctuations large enough to hinder reproducible measurements of the stress field inside the pile [4, 5], and even suspected to be the driving factor leading to large-scale "static avalanches" [6]. Indeed, the slow relaxation and compaction of a granular material, which has been hitherto produced by the input of mechanical energy [7], can be induced by periodically raising and then lowering the temperature of the granular assembly, as recently brought to the fore by Chen and co-workers [8]. However, the compaction dynamics as well as the basic mechanisms at stake remain unknown. In this letter, we address the following questions: What is the dynamic of the top of a granular column submitted to thermal cycling? Does this compaction process exhibit features analogous to aging, as other compaction processes (tapping, cyclic shear...) do? And finally, what is the behaviour of the grain assembly in the limit of low amplitude temperature-cycles, i.e. well below a cycling amplitude of 40°C [8]?

Compaction of granular materials has been thoroughly studied over the past 20 years both experimentally and theoretically [7]. It has been achieved by direct input of mechanical energy. By far, the most-used device consists of a vertical column which is periodically shaken with a controlled vibration intensity. Under vibration, a low-density granular-pile compacts towards a steady state with a final density that depends on the intensity of the vibrations [1, 2]. Nonetheless, those final states have been shown not to depend on the initial conditions (packing fraction) and even to be genuine thermodynamic states [9]. In addition to their general interest [7], vibrated granular materials have been found to exhibit deep common physical features with out-of-equilibrium systems, such as glasses. First, the spectrum of density fluctuations was found to be strongly non-Lorentzian [10], which is a signature of multiple timescales in the system. Also, the response of the column to an abrupt change in shaking intensity is analogous to aging in glasses [11, 12]. Finally, in the limit of small-intensity vibrations, the relaxation rate becomes so slow that the system cannot reach the steady-state density within the experiment timescale. At the microscopic scale, a grain exhibits random motion within confined volumes or cages, occasionally a longer exploration corresponding to a change of cage, and rarely jumps, during which it moves "significantly" [13]. Cyclic shear deformations of a granular assembly also lead to compaction and present the transient cage effects reminding one of colloidal glasses [3, 14]. Those experimental results have motivated numerous theoretical and numerical works, a review of which can be found in [7].

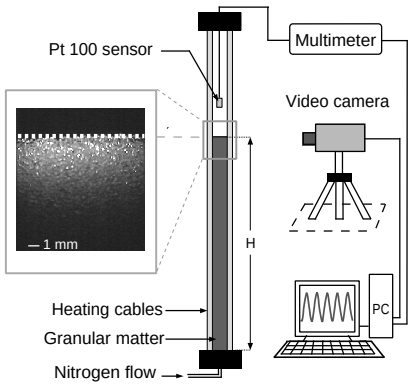

FIG. 1: **Sketch of the experimental setup.** Inset: picture of the upper part of the column. The granular level is indicated by the white dotted-line.

Up to now, the heating of a granular pile has only been used to develop dynamic light-scattering experiments [15] or to probe numerically its impact on the force network [16]. Here, we report the time-resolved compaction dynamics of a granular column submitted to a large number of temperature cycles and shed new lights on the key ingredients governing this process.

Experimental set-up - The experimental set-up (Fig. 1) consists in a vertical glass tube (height 1.7 m, inner diameter 13 mm) firmly fastened to a wall into the basement of the physics department, in order to flee mechanical vibrations. The sample consists of a column (height H) of spherical glass beads (diameter d) poured into the tube. An additional gas-input, at the bottom of the column, makes it possible to loosen the pile thanks to an upward flow of dry nitrogen. The temperature cycles are imposed by means of a heating cable (Prolabo, 40 W/m) directly taped on the outer surface of the tube wall. The resulting temperature is measured by means of a sensor (Pt100, located close to the free surface of the granular material) and a multimeter (Keithley Intruments, Model 196). The free surface of the material, which is illuminated by a red LED (Kingbright, L-793SRC-E, located inside the tube, above the granular material) is imaged from the side with a video camera (Panasonic, WV-BP500) connected to a frame grabber board (Data Translation, DT2255). A macro, running under a data-processing software (WaveMetrics, IGOR Pro 4.0), makes it possible to drive the heating power, to record the resulting variations of the temperature and to measure accurately the height H from the images: A subpixel resolution (namely, less than a tenth of a pixel which typically stands for 5 μm) is achieved by considering the average position of the free surface, assumed to correspond to the inflection point in the vertical intensity-profile averaged over the whole diameter of the tube. Measurements are performed 20 times per temperature cycle.

Due to long experimental times, we limit our report to a given diameter $d = 510 \pm 90$ μm of the grains (Matrasur Corp.) and to a given period $2\pi/\omega = 600$ s of the cycles. The cycling period, 10 minutes, is arbitrarily chosen to be small enough to avoid excessively-long experimental times but large enough to insure that the associated thermal penetration-length $l_p \equiv \sqrt{2\lambda/(\rho C \omega)} \simeq 6$ mm is about the tube radius ($\lambda \simeq 0.2$ W m^{-1} K^{-1} and $C \simeq 10^6$ J m^{-3} K^{-1} respectively denote the thermal conductivity and heat capacity of a typical glass-grains pile [17].) It is here crucial to note that the column is heated homogeneously along its whole length but that the temperature is likely to vary in the radial direction.

Prior to each experiment, the granular column is prepared in a low-density state thanks to the dry-nitrogen upward flow. The top of the column is then higher than the field imaged by the camera (typically 1 cm above) and we set the amplitude of the cycles, ΔT, to the largest accessible value, $\Delta T = 27.1°$ C. The preparation of the sample ends when the top of the column enters the observation field. At this point, the granular column is "quenched": The amplitude of the cycles is set to the chosen value ΔT lying bewteen 0 and 27.1°C, which defines the origin of time $t = 0$. The granular column is subsequently submitted to at least 1000 cycles (7 days).

First experimental observations - Under the action of the temperature cycles the column height decreases: we report the variation $h_n \equiv H(2\pi n/\omega) - H(0)$, where n denotes the time in units of the cycle period or, equivalently when integer, the number of imposed cycles (Fig. 2). We observe that the thermal-induced compaction is a very slow phenomenon: after 7 days (1000 cycles), the decrease of the height is of about 1.5 cm (about 1% of the height H), which indicates that the system remains very far from the maximum compaction (roughly a decrease of about 10% of the column height, i.e. $h_n \sim 10$ cm) within the experimental time. Accordingly, after the experiment, we checked that a single finger tap produces a collapse of the granular height of one centimeter at a rough estimate. We also checked during 3 days that the height H of the column stays constant when no temperature cycles are imposed, which proves that ambient mechanical vibrations and changes in the room temperature have no (or little) effect in our experimental conditions.

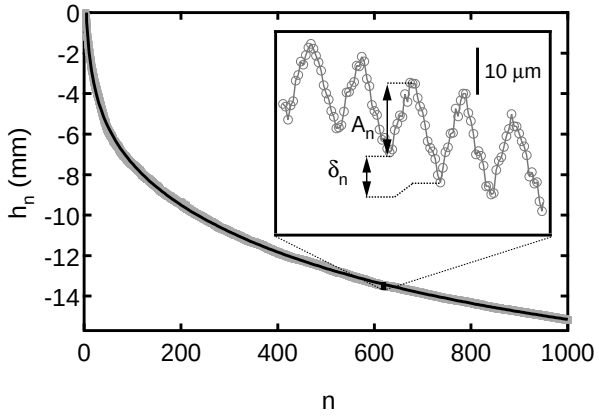

FIG. 2: **Height variation h_n vs. number of cycles n.** One observes first an exponential behavior at short time followed by a subsequent logarithmic creep at long time (The black curve corresponds to the test function h_n^t defined in the text.) Inset: Oscillations of the column height associated with the temperature cycles (A_n and δ_n are respectively defined to be the amplitude of the increase and the drift of h_n at the cycle n.) For the chosen ΔT, the column settles slightly at each cycle ($H = 140$ cm, $2\pi/\omega = 600$ s and $\Delta T = 10.8°$C.)

Before we discuss the compaction dynamics, it is relevant to determine which part of the column is involved in the phenomenon. Our measurements are accurate enough for observing oscillations of H associated with the temperature variations (Fig. 2, inset). We observe on the raw data that the amplitude A_n, which is proportional to ΔT, increases logarithmically with n (Fig. 3). The oscillations of H are due to the thermal dilation of both the tube and the granular material. In order to assess the contribution of the granular material, we first determine the amplitude, $\delta h_t(z)$, of the tube displacement in the laboratory frame as a function of the height z (origin at the bottom of the column) by marking its

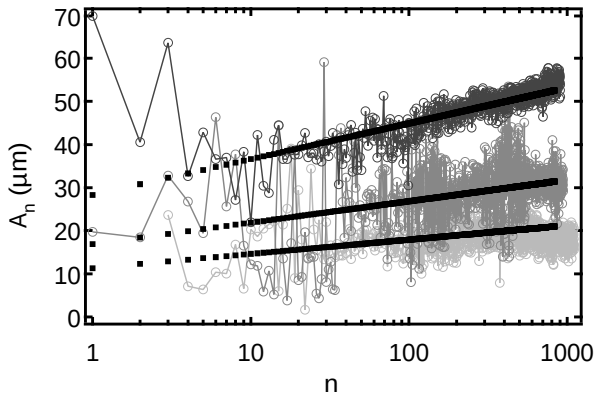

FIG. 3: **Amplitude A_n vs. number of cycles n.** The data are successfully accounted for by $A_n = \Delta T\,[a_0 + b_0\,\ln(n)]$ with $a_0 \simeq 13.8$ nm/K and $b_0 \simeq 2.5$ nm/K ($H = 140$ cm, $2\pi/\omega = 600$ s and, from bottom to top, $\Delta T = 10.8, 16.2$ and 27.1°C.)

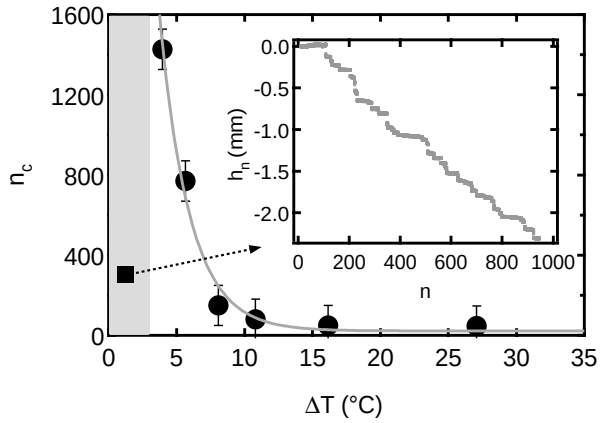

FIG. 4: **Characteristic number n_c vs. amplitude ΔT.** The characteristic number of cycles n_c increases drastically when ΔT is decreased and even seems to diverge for $\Delta T_c \simeq 3$°C. Inset: For $\Delta T < \Delta T_c$, the column settles, by jumps, linearly with time. For readibility, we only display data obtained during the first half of the experiment duration (14 days, $H = 140$ cm, $2\pi/\omega = 600$ s and $\Delta T = 2.8$°C.)

outer wall. The amplitude $\delta h_t(z)$ is found to be linear in z and the slope provides us with an estimate of the linear thermal-expansion coefficient, $\kappa = (3.6\pm0.4)\times10^{-6}$ K^{-1}, of the tube material. Then, considering the relative variation of the inner volume, we write the relation between the amplitude A_n and the relative variation, $\delta V_g/V_g$, of the volume V_g of the granular material: $A_n - \delta h_t(0) = H(\delta V_g/V_g - 2\kappa\Delta T)$. Experiments performed for different height H demonstrate that $A_n - \delta h_t(0)$ is proportional to H, which shows that $\delta V_g/V_g$ is independent of H. We thus infer that *the whole height H of the granular column is involved in the observed oscillations of the free surface.*

Compaction dynamics - For a large ΔT (typically more than 3°C), the column systematically compacts during the decrease of the temperature, at each cycle, and the amplitude δ_n of the collapses (defined in Fig. 2, inset) decreases when n increases, which accounts for the overall behavior of H as a function of n. We show here that the results obtained by Chen and co-workers for $\Delta T > 40$°C [8] hold true at much smaller ΔT. Besides, we resolve the column dynamics and, this, over a significant longer duration. Here, the column height first decreases exponentially to reach, after the characteristic number n_c of cycles, a logarithmic behavior at long time (Fig. 2). This response to the thermal quenching is very similar to the one the system exhibits to step strain solicitation [18]. We estimated n_c from the experimental data by interpolating h_n with the test function $h_n^t \equiv h_0 + h_e\,\exp(-n/n_c) + h_l\,\ln(n)$. In addition, we checked that the shortest characteristic time obtained from the interpolation by a double exponential [19] is consistent with n_c and presents the same behavior as a function of ΔT. We observe that n_c drastically increases when ΔT is decreased. Moreover, n_c even seems to diverge at ΔT_c (We get $2.8 < \Delta T_c < 4.0$ °C, Fig. 4), which prefigures a qualitatively-different dynamics of the column compaction at a lower amplitude.

For $\Delta T < \Delta T_c$, one indeed observes that the column is not flowing regularly anymore, but evolves by successive collapses separated by rest periods (Fig. 4, inset). The overall compaction velocity is constant (steady regime) and it is therefore relevant to consider the probability distribution $P(\Delta n)$ of the number of cycles Δn between two successive collapses (Fig. 5). However, due to the small number of collapses, $N = 175$, within the experimental time (14 days), the behavior of $P(\Delta n)$ is difficult to assess directly and we consider the cumulated probability, $F(\Delta n) \equiv \int_{\Delta n}^{\infty} P(x)\,\mathrm{d}x$, instead [20]. We obtain $F(\Delta n) \simeq \exp(-\Delta n/\Delta n^*)$ with $\Delta n^* \simeq 12$ for $\Delta T = 2.8$°C (Fig. 5a). The delay Δn between two successive collapses is thus observed to be exponentially distributed. By contrast, the amplitude of the corresponding collapses, δ_n, exhibits a Gaussian probability-distribution $Q(\delta)$ whose width is estimated to be $\delta^* = 63 \pm 4\,\mu$m (Fig. 5b), roughly a tenth of the grain diameter. The compaction of the material is thus shown to result from collapses that are randomly distributed in time.

Discussion and conclusion - The mechanisms leading to the compaction could be either extrinsic (the dilation of the container at stake) or intrinsic (only due to the dilation of the grains.) Chen *et al.* suggested that the compaction could be due to "the difference between the thermal expansion of the container and of the grains" [8], thus to an extrinsic mechanism. In our case, the beads ($\kappa_b = 9\times10^{-6}$ K^{-1}) and the container ($\kappa = 3.6\times10^{-6}$ K^{-1}) are made of glass and present roughly the same thermal expansion coefficient [21]. Thus, we do not think that the thermal dilation of the container is the primary cause of the compaction. This conclusion is supported

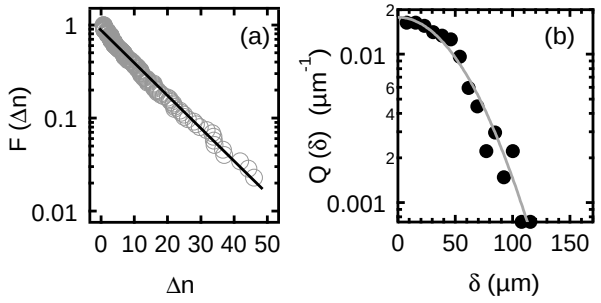

FIG. 5: **Statistics below the transition**. (a) : The cumulated probability $F(\Delta n) \simeq \exp(-\Delta n/\Delta n^*)$ is an exponential function of Δn and the characteristic number $\Delta n^* \simeq 12$. (b) : The probability distribution $Q(\delta) \simeq \exp[-(\delta/\delta^*)^2]$ is a Gaussian function whose width $\delta^* = 63 \pm 4\,\mu$m. The statistical data reported herein correspond to the 175 collapses observed within the experimental time (14 days, $H = 140$ cm, $2\pi/\omega = 600$ s and $\Delta T = 2.8°$C).

by preliminary results obtained by heating periodically the granular column along its axis [22] and the recent simulations by Vargas and McCarthy [16]. Dealing with an intrinsic mechanism, we guess that an overall temperature change would only produce a homogeneous dilation of the pile and no spatial reorganisation of the grains. We thus propose the following scenario, compatible with previous results [8]: due to the radial thermal-gradient of amplitude $\Delta T/l_p$, the grains are subjected to a periodic shear, γ, along the vertical which induces local rearrangments and, accidentally, internal avalanches. This mechanism is supported by the estimate of the shear $\gamma = \kappa_b(\Delta T/l_p)L$, where L is a characteristic size which ranges from the grain- to the tube-diameter. We obtain, for $L \in [0.510, 13]$ mm, that the relative displacement of two neighbor grains $\Gamma \equiv \gamma d/\Delta T \in [0.4, 10]$ nm/°C whereas, using an AFM, we estimated the maximum size of the asperities associated with the surface roughness of the beads, $s \simeq 100$ nm (Invoking the surface roughness is supported by numerical simulations: for perfectly smooth grains the maximum compacity is reached after a few cycles [16], which contrasts with our observations.) In this framework, the continuous compaction would thus correspond to amplitudes ΔT such as $\Gamma\Delta T > s$, whereas jumps would be observed for $\Delta T < \Delta T_c \simeq s/\Gamma$. From the experimental $\Delta T_c \simeq 3°$C, we could infer that the typical length at stake, L, is about the tube radius and, thus, the transition a finite-size effect. Due to long experimental times, we limited our report to results which clearly demonstrate the efficiency of the techniques. The dependance of the critical amplitude ΔT_c on the column height and diameter as well as on the cycling frequency and grain roughness is necessary to corroborate or discard the proposed mechanism. In particular, the sensitivity to the frequency ω will indicate the role played by the temperature gradient. We also plan to locate the origin of the collapses by listening to the associated noise and to observe the local displacement by optical means. This way, we hope to correlate the macroscopic behavior to the local dynamics as already done for tapping [13, 23] and cycling shear experiments [3, 24].

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[1] J.B. Knight *et al.*, Phys. Rev. E **51** 3957 (1995).
[2] P. Philippe and D. Bideau, Eur. Phys. Lett. **60**, 677 (2002).
[3] O. Pouliquen *et al.* Phys. Rev. Lett. **91**, 014301 (2003).
[4] L. Vanel and E. Clément, Eur. Phys. J. B **11**, 525 (1999).
[5] E. Clément, *et al. Proceedings of the IIIrd Intern. Conf. on Powders & Grains* (Balkema, Rotterdam, 1997).
[6] P. Claudin and J.-P. Bouchaud, Phys. Rev. Lett. **78**, 231 (1997).
[7] P. Richard *et al.*, Nature Materials, **4**, 121 (2005).
[8] K. Chen *et al.*, Nature **442**, 257 (2006).
[9] P. Ribière *et al.*, Eur. Phys. J. E **22**, 249 (2007).
[10] E.R. Nowak *et al.*, Phys. Rev. E **57**, 1971 (1998).
[11] C. Josserand *al.* Phys. Rev. Lett. **85** 3632 (2000).
[12] A. Kabla and G. Debregeas, Phys. Rev. Lett. **92**, 035501 (2004).
[13] P. Ribière *et al.*, Phys. Rev. Lett. **95**, 268001 (2005).
[14] M. Nicolas *et al.*, Eur. Phys. J. E **3**, 309 (2000).
[15] L. Djaoui and J. Crassous, Gran. Matt. **7**, 1985 (2005).
[16] W.L. Vargas and J.J. McCarthy, AiChE Journal **47**, 1052 (2001); Phys. Rev. E **76**, 041301 (2007).
[17] J.-C. Géminard and H. Gayvallet, Phys. Rev. E **64**, 041301 (2001).
[18] J. Brujić *et al.*, Phys. Rev. Lett. **95**, 128001 (2005).
[19] G.C. Barker and A. Mehta, Phys. Rev. E **47**, 184 (1993).
[20] A convenient way to estimate $F(\Delta n)$ is to re-order the set $\{\Delta n^{(i)}\}$ of the N-1 experimental values into the series $\{\Delta n'^{(j)}\}$ such that $\Delta n'^{(1)} > \Delta n'^{(2)} > \ldots > \Delta n'^{(N-1)}$. Plotting $j/(N\text{-}1)$ versus $\Delta n'^{(j)}$ for $j \in [1, N\text{-}1]$ gives a rather precise estimate of $F(\Delta n)$. See for instance: B.A. Berg and R.C. Harris, arXiv:0712.3852v1 (2007).
[21] In [8], κ/κ_b and κ_b/κ are systematically larger than 13.
[22] J.-C. Géminard, *Habilitation à Diriger des Recherches*, Université Joseph Fourier, Grenoble I, p. 32 (2003) *http://tel.archives-ouvertes.fr/tel-00294761/fr/*
[23] G. Lumay and N. Vandewalle, Phys. Rev. Lett. **95**, 028002 (2005).
[24] G. Marty and O. Dauchot, Phys. Rev. Lett. **94**, 015701 (2005).